The semi-inclusive DIS the polarized leptons on the polarized nucleons

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$$\ell + N \to \nu + h + X$$

Differential cross sections of the process (1) in the case of a lepton:

$$\left(\frac{d^3\sigma_{\ell^-}}{dxdydz}\right)^h = 2\rho x\left\{\sum_{q_i,q_j} q_i(x,Q^2)D_{q_j}^h(z,Q^2) + y_1^2\sum_{\bar{q}_j,\bar{q}_i}\bar{q}_j(x,Q^2)D_{\bar{q}_i}^h(z,Q^2) + \right.$$

$$\left. + P_N\left(\sum_{q_i,q_j}\Delta q_i(x,Q^2)D_{q_j}^h(z,Q^2) - y_1^2\sum_{\bar{q}_j,\bar{q}_i}\Delta\bar{q}_j(x,Q^2)D_{\bar{q}_i}^h(z,Q^2)\right)\right\},$$

where $q_i = d,s,b$, $q_j = u,c,t$, $\bar{q}_i = \bar{d},\bar{s},\bar{b}, \bar{q}_j = \bar{u},\bar{c},\bar{t}$;

for antilepton

$$\left(\frac{d^3\sigma_{\ell^+}}{dxdydz}\right)^h = 2\rho x\left\{y_1^2\sum_{q_i,q_j} q_i(x,Q^2)D_{q_j}^h(z,Q^2) + \sum_{\bar{q}_j,\bar{q}_i}\bar{q}_j(x,Q^2)D_{\bar{q}_i}^h(z,Q^2) + \right.$$

$$\left. + P_N\left(\sum_{q_i,q_j}\Delta q_i(x,Q^2)D_{q_j}^h(z,Q^2) - \sum_{\bar{q}_j,\bar{q}_i}\Delta\bar{q}_j(x,Q^2)D_{\bar{q}_i}^h(z,Q^2)\right)\right\},$$

where $q_i = u,c,t$, $q_j = d,s,b$, $\bar{q}_i = \bar{u},\bar{c},\bar{t}$, $\bar{q}_j = \bar{d},\bar{s},\bar{b}$.

Here

$$\rho = \frac{G^2 s}{2\pi} \cdot \left(\frac{m_w^2}{m_w^2 + Q^2} \right)^2 ,$$

$$y_1 = 1 - y ,$$

G is Fermi constant, m_w is the W-boson mass,

$$x = \frac{Q^2}{2p \cdot q}, \quad y = \frac{p \cdot q}{p \cdot k},$$

$$Q^2 = -q^2 = -(k - k')^2 ,$$

$s = 2p \cdot k$, $k(k')$ and p are the initial (final) lepton and proton 4-momenta respectively, P_N is the degree of longitudinal polarization of proton, $q(x,Q^2)(\Delta q(x,Q^2))/\bar{q}(x,Q^2)(\Delta\bar{q}(x,Q^2))$ are the unpolarized (polarized) quark/antiquark distribution functions, $D_q^{\ell^-}(z,Q^2)(D_q^{\ell^+}(z,Q^2)$ are the fragmentation functions of quark (antiquark) with flavor q to the hadron h.

$$A_{\ell^-}^{\pi^+-\pi^-}=\frac{\left(d\sigma_{\ell^-d}^{\downarrow\uparrow}\right)^{\pi^+-\pi^-}-\left(d\sigma_{\ell^-d}^{\downarrow\downarrow}\right)^{\pi^+-\pi^-}}{\left(d\sigma_{\ell^-d}^{\downarrow\uparrow}\right)^{\pi^+-\pi^-}+\left(d\sigma_{\ell^-d}^{\downarrow\downarrow}\right)^{\pi^+-\pi^-}},$$

$$A_{\ell^+}^{h^+-h^-}=\frac{\left(d\sigma_{\ell^+d}^{\uparrow\uparrow}\right)^{\pi^+-\pi^-}-\left(d\sigma_{\ell^+d}^{\uparrow\downarrow}\right)^{\pi^+-\pi^-}}{\left(d\sigma_{\ell^+d}^{\uparrow\uparrow}\right)^{\pi^+-\pi^-}+\left(d\sigma_{\ell^+d}^{\uparrow\downarrow}\right)^{\pi^+-\pi^-}},$$

$$A_{\pm}^{\pi^+-\pi^-}=\frac{\left[\left(d\sigma_{\ell^-d}^{\downarrow\uparrow}\right)^{\pi^+-\pi^-}\pm\left(d\sigma_{\ell^+d}^{\uparrow\uparrow}\right)^{\pi^+-\pi^-}\right]-\left[\left(d\sigma_{\ell^-d}^{\downarrow\downarrow}\right)^{\pi^+-\pi^-}\pm\left(d\sigma_{\ell^+d}^{\uparrow\downarrow}\right)^{\pi^+-\pi^-}\right]}{\left[\left(d\sigma_{\ell^-d}^{\downarrow\uparrow}\right)^{\pi^+-\pi^-}\pm\left(d\sigma_{\ell^+d}^{\uparrow\uparrow}\right)^{\pi^+-\pi^-}\right]+\left[\left(d\sigma_{\ell^-d}^{\downarrow\downarrow}\right)^{\pi^+-\pi^-}\pm\left(d\sigma_{\ell^+d}^{\uparrow\downarrow}\right)^{\pi^+-\pi^-}\right]},$$

где $d\sigma=\dfrac{d^3\sigma}{dxdydz}$, $d\sigma^{\pi^+-\pi^-}=d\sigma^{\pi^+}-d\sigma^{\pi^-}$.

The case of $\pi -$ meson production

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for proton target

$$A_{\ell^-}^{\pi^+-\pi^-} = \frac{\Delta u(x,Q^2) - y_1^2 \Delta \bar{d}(x,Q^2)}{u(x,Q^2) + y_1^2 \bar{d}(x,Q^2)},$$

$$A_{\ell^+}^{\pi^+-\pi^-} = \frac{y_1^2 \Delta d(x,Q^2) - \Delta \bar{u}(x,Q^2)}{y_1^2 d(x,Q^2) + \bar{u}(x,Q^2)},$$

$$A_{+,p}^{\pi^+-\pi^-} = \frac{\Delta u(x,Q^2) + \Delta \bar{u}(x,Q^2) - y_1^2 \left(\Delta d(x,Q^2) + \Delta \bar{d}(x,Q^2)\right)}{u_V(x,Q^2) - y_1^2 d_V(x,Q^2)},$$

$$A_{-,p}^{\pi^+-\pi^-} = \frac{\Delta u_V(x,Q^2) + y_1^2 \Delta d_V(x,Q^2)}{u(x,Q^2) + \bar{u}(x,Q^2) + y_1^2 \left(d(x,Q^2) + \bar{d}(x,Q^2)\right)},$$

$y \to 0$

$$A_{\ell^- p}^{\pi^+ - \pi^-} = \frac{\Delta u(x,Q^2) - \Delta \bar{d}(x,Q^2)}{u(x,Q^2) + \bar{d}(x,Q^2)}$$

$$A_{\ell^+ p}^{\pi^+ - \pi^-} = \frac{\Delta d(x,Q^2) - \Delta \bar{u}(x,Q^2)}{d(x,Q^2) + \bar{u}(x,Q^2)}$$

$$A_{+,p}^{\pi^+ - \pi^-} = \frac{\Delta u(x,Q^2) + \Delta \bar{u}(x,Q^2) - \left(\Delta d(x,Q^2) + \Delta \bar{d}(x,Q^2)\right)}{u_V(x,Q^2) - d_V(x,Q^2)}$$

$$A_{-,p}^{\pi^+ - \pi^-} = \frac{\Delta u_V(x,Q^2) + \Delta d_V(x,Q^2)}{u(x,Q^2) + \bar{u}(x,Q^2) + d(x,Q^2) + \bar{d}(x,Q^2)}$$

$y \to 0$

$$A_{\ell^- n}^{\pi^+ - \pi^-} = \frac{\Delta d(x,Q^2) - \Delta \bar{u}(x,Q^2)}{d(x,Q^2) + \bar{u}(x,Q^2)}$$

$$A_{\ell^+ n}^{\pi^+ - \pi^-} = \frac{\Delta u(x,Q^2) - \Delta \bar{d}(x,Q^2)}{u(x,Q^2) + \bar{d}(x,Q^2)}$$

$$A_{+,n}^{\pi^+ - \pi^-} = \frac{\Delta d(x,Q^2) + \Delta \bar{d}(x,Q^2) - \left(\Delta u(x,Q^2) + \Delta \bar{u}(x,Q^2)\right)}{d_V(x,Q^2) - u_V(x,Q^2)}$$

$$A_{-,n}^{\pi^+ - \pi^-} = \frac{\Delta d_V(x,Q^2) + \Delta u_V(x,Q^2)}{d\left(x,Q^2\right) + \bar{d}\left(x,Q^2\right) + u(x,Q^2) + \bar{u}(x,Q^2)}$$

$$y \to 1$$

$$A^{\pi^+-\pi^-}_{\ell^- p} = \frac{\Delta u(x,Q^2)}{u(x,Q^2)} \qquad A^{\pi^+-\pi^-}_{+,p} = \frac{\Delta u(x,Q^2) + \Delta \bar{u}(x,Q^2)}{u_V(x,Q^2)}$$

$$A^{\pi^+-\pi^-}_{\ell^+ p} = \frac{-\Delta \bar{u}(x,Q^2)}{\bar{u}(x,Q^2)} \qquad A^{\pi^+-\pi^-}_{-,p} = \frac{\Delta u_V(x,Q^2)}{u(x,Q^2) + \bar{u}(x,Q^2)}$$

$$A^{\pi^+-\pi^-}_{\ell^- n} = \frac{\Delta d(x,Q^2)}{d(x,Q^2)} \qquad A^{\pi^+-\pi^-}_{+,n} = \frac{\Delta d(x,Q^2) + \Delta \bar{d}(x,Q^2)}{d_V(x,Q^2)}$$

$$A^{\pi^+-\pi^-}_{\ell^+ n} = \frac{-\Delta \bar{d}(x,Q^2)}{\bar{d}(x,Q^2)} \qquad A^{\pi^+-\pi^-}_{-,n} = \frac{\Delta d_V(x,Q^2)}{d\left(x,Q^2\right) + \bar{d}\left(x,Q^2\right)}$$

$$y=\frac{1}{2}$$

$$A_{\ell^- p}^{\pi^+-\pi^-}=\frac{\Delta u(x,Q^2)-0.25\Delta\bar{d}(x,Q^2)}{u(x,Q^2)+0.25\bar{d}(x,Q^2)}$$

$$A_{\ell^+ p}^{\pi^+-\pi^-}=\frac{0.25\Delta d(x,Q^2)-\Delta\bar{u}(x,Q^2)}{0.25d(x,Q^2)+\bar{u}(x,Q^2)}$$

$$A_{+,p}^{\pi^+-\pi^-}=\frac{\Delta u(x,Q^2)+\Delta\bar{u}(x,Q^2)-0.25\left(\Delta d(x,Q^2)+\Delta\bar{d}(x,Q^2)\right)}{u_V(x,Q^2)-0.25\left(d(x,Q^2)+\bar{d}(x,Q^2)\right)}$$

$$A_{-,p}^{\pi^+-\pi^-}=\frac{\Delta u_V(x,Q^2)+0.25\Delta d_V(x,Q^2)}{u(x,Q^2)+\bar{u}(x,Q^2)+0.25\left(d(x,Q^2)+\bar{d}(x,Q^2)\right)}$$

$$y = \frac{1}{2}$$

$$A_{\ell^- n}^{\pi^+ - \pi^-} = \frac{\Delta d(x,Q^2) - 0.25\Delta\bar{u}(x,Q^2)}{d(x,Q^2) + 0.25\bar{u}(x,Q^2)}$$

$$A_{\ell^+ n}^{\pi^+ - \pi^-} = \frac{0.25\Delta u(x,Q^2) - \Delta\bar{d}(x,Q^2)}{0.25u(x,Q^2) + \bar{d}(x,Q^2)}$$

$$A_{+,n}^{\pi^+ - \pi^-} = \frac{\Delta d(x,Q^2) + \Delta\bar{d}(x,Q^2) - 0.25\left(\Delta u(x,Q^2) + \Delta\bar{u}(x,Q^2)\right)}{d_V(x,Q^2) - 0.25u_V(x,Q^2)}$$

$$A_{-,n}^{\pi^+ - \pi^-} = \frac{\Delta d_V(x,Q^2) + 0.25\Delta u_V(x,Q^2)}{u(x,Q^2) + \bar{u}(x,Q^2) + 0.25\left(d\left(x,Q^2\right) + \bar{d}\left(x,Q^2\right)\right)}$$

$$A^{\pi^+-\pi^-}_{+,p}\,,\ A^{\pi^+-\pi^-}_{+,n} \Longrightarrow \begin{array}{l} \Delta u(x,Q^2)+\Delta\bar{u}(x,Q^2) \\ \Delta d(x,Q^2)+\Delta\bar{d}(x,Q^2) \end{array}$$

$$A^{\pi^+-\pi^-}_{\ell^-,p}\,,\ A^{\pi^+-\pi^-}_{\ell^+,n} \Longrightarrow \begin{array}{l} \Delta\bar{d}(x,Q^2) \\ \Delta u(x,Q^2) \end{array}$$

$$A^{\pi^+-\pi^-}_{\ell^+,p}\,,\ A^{\pi^+-\pi^-}_{\ell^-,n} \Longrightarrow \begin{array}{l} \Delta\bar{u}(x,Q^2) \\ \Delta d(x,Q^2) \end{array}$$

$$A^{\pi^+-\pi^-}_{-,p}\,,\ A^{\pi^+-\pi^-}_{-,n} \Longrightarrow \begin{array}{l} \Delta u_V(x,Q^2) \\ \Delta d_V(x,Q^2) \end{array}$$

The ways to get spin information

For the analysis of nucleon spin structure we introduce the first moments of parton distributions as follows

$$\Delta q(Q^2) = \int_0^1 \Delta q(x, Q^2)\, dx,$$

$$\Delta \overline{q}(Q^2) = \int_0^1 \Delta \overline{q}(x, Q^2)\, dx,$$

which correspond to the quark q (antiquark $\overline{q}$) contributions to the spin of nucleon.

Numerical results

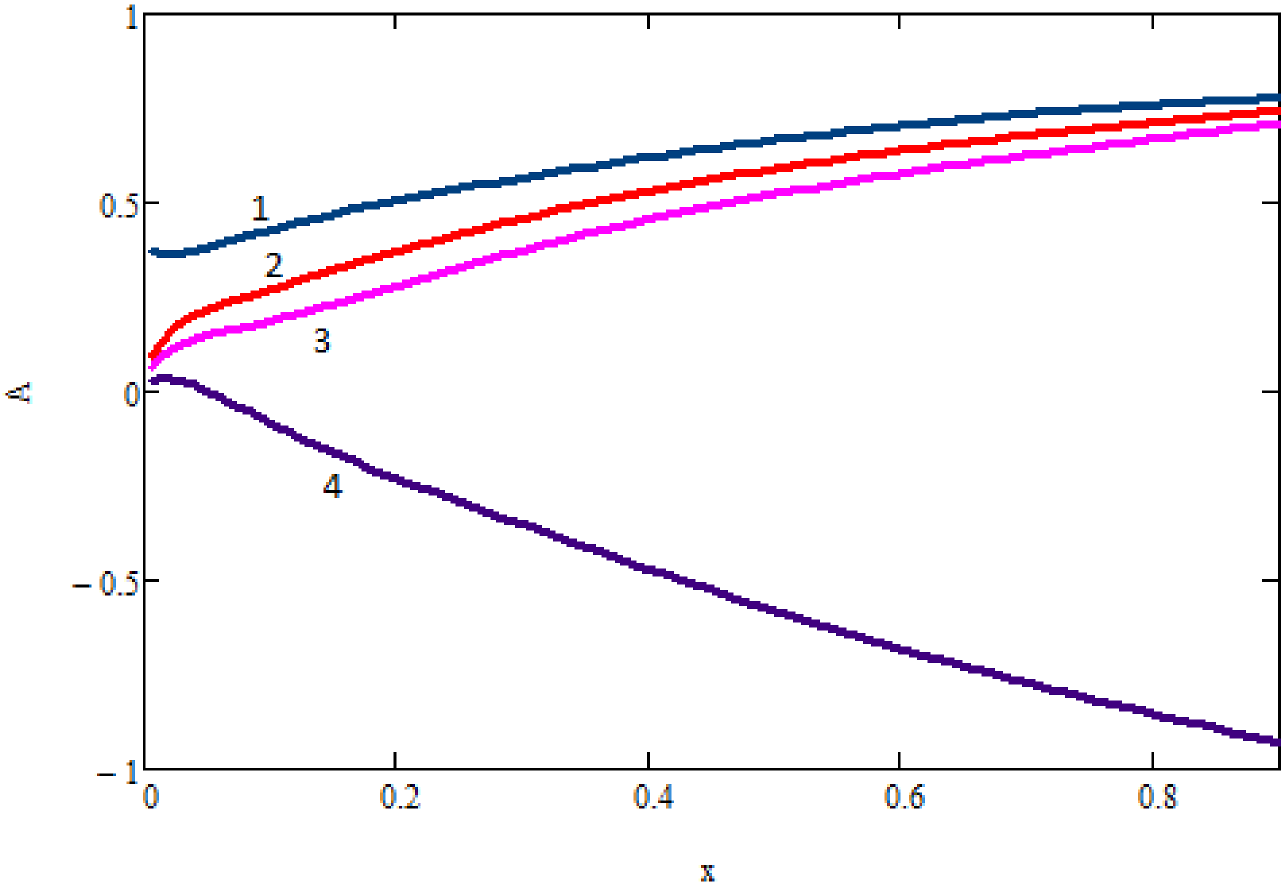

Figure 1. Asymmetry: $A_{+,p}^{\pi^+-\pi^-}$ (1), $A_{\ell^- p}^{\pi^+-\pi^-}$ (2), $A_{-,p}^{\pi^+-\pi^-}$ (3), $A_{\ell^+ p}^{\pi^+-\pi^-}$ (4) for $y = \frac{1}{2}$.

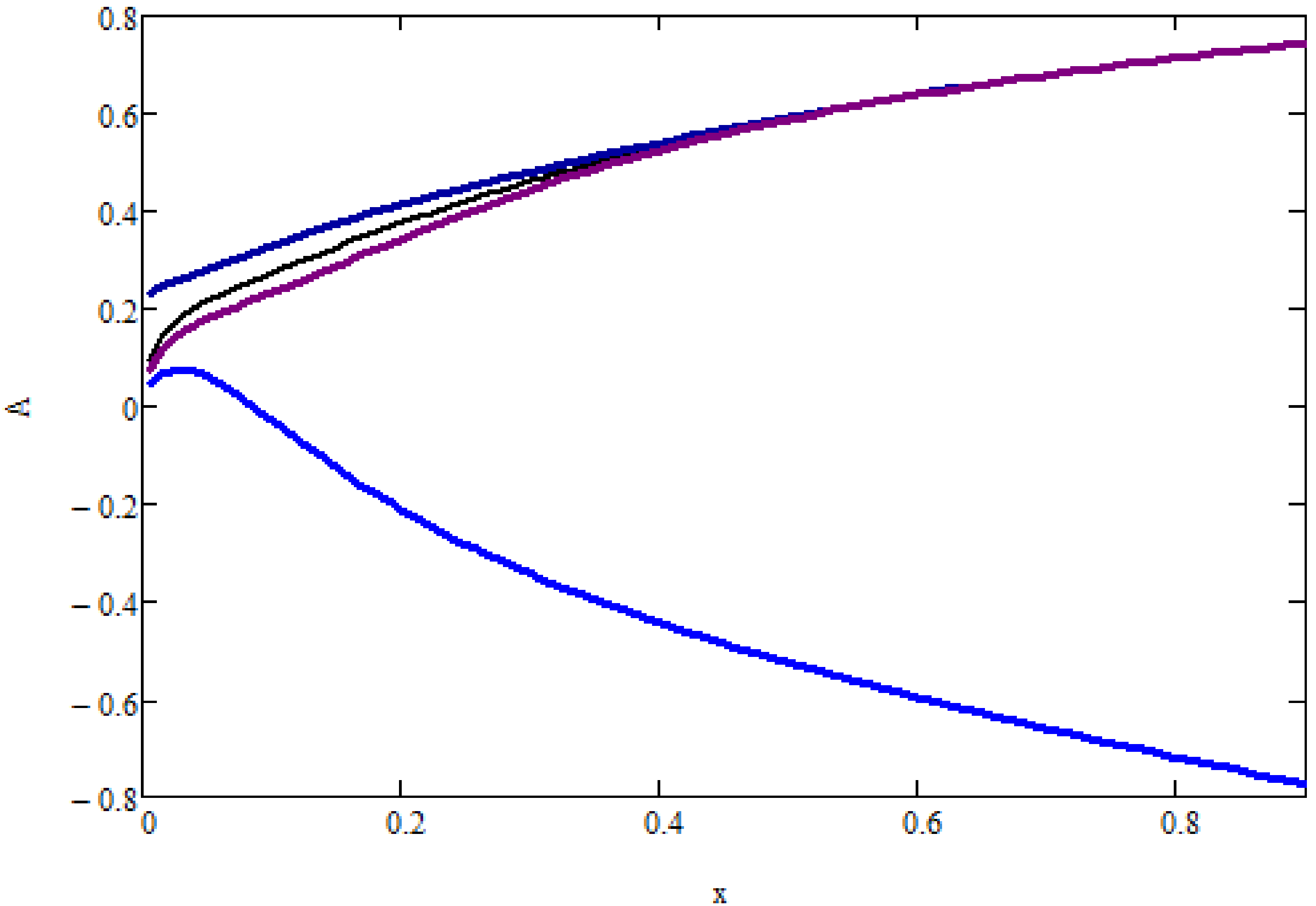

Figure 2. Asymmetry: $A_{+,p}^{\pi^+-\pi^-}$, $A_{\ell^-p}^{\pi^+-\pi^-}$, $A_{-,p}^{\pi^+-\pi^-}$, $A_{\ell^+p}^{\pi^+-\pi^-}$ for $y=1$.

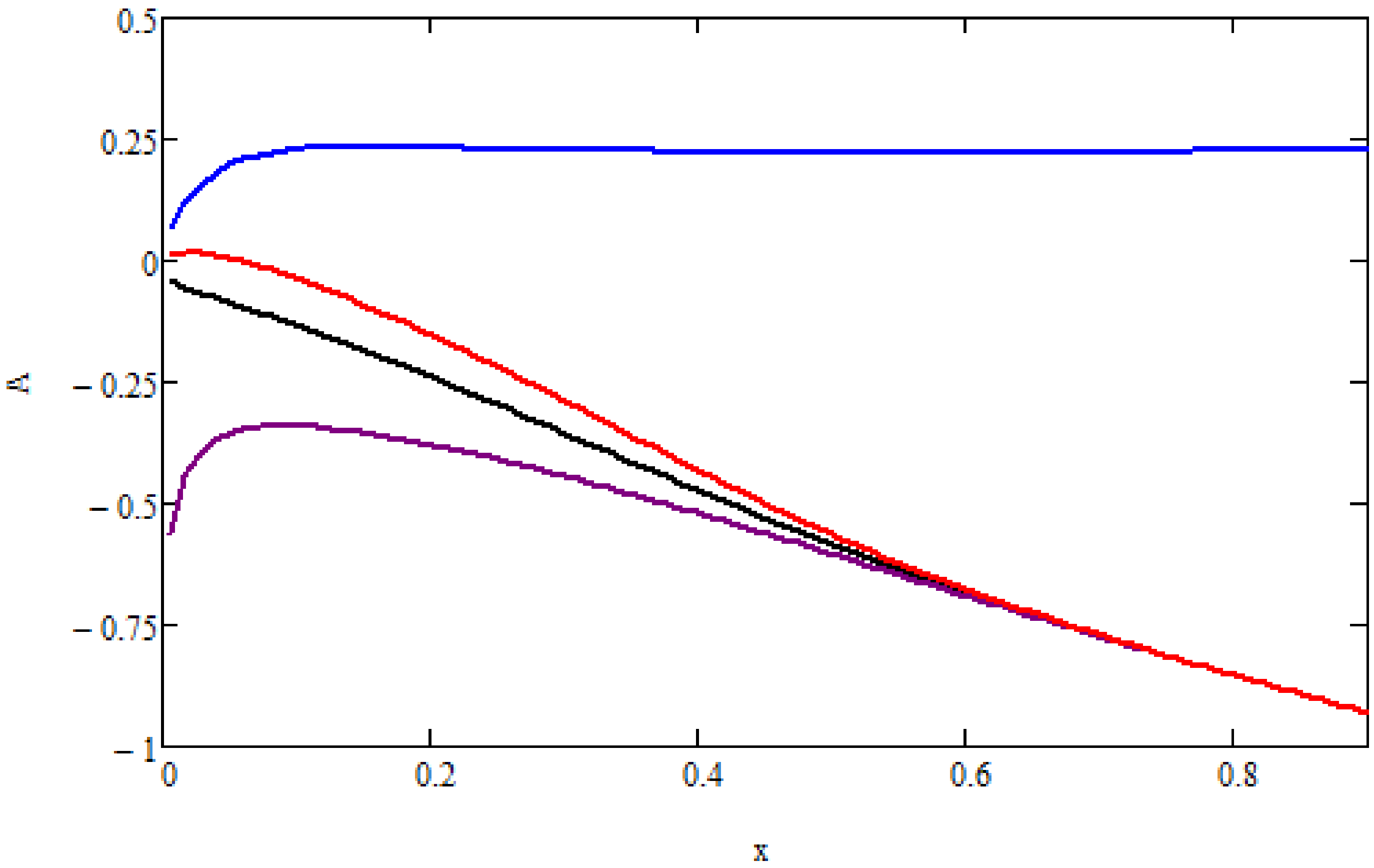

Figure 3. Asymmetry: $A_{\ell^+ n}^{\pi^+ - \pi^-}$, $A_{-,n}^{\pi^+ - \pi^-}$, $A_{\ell^- n}^{\pi^+ - \pi^-}$, $A_{+,n}^{\pi^+ - \pi^-}$, for $y = 1$.

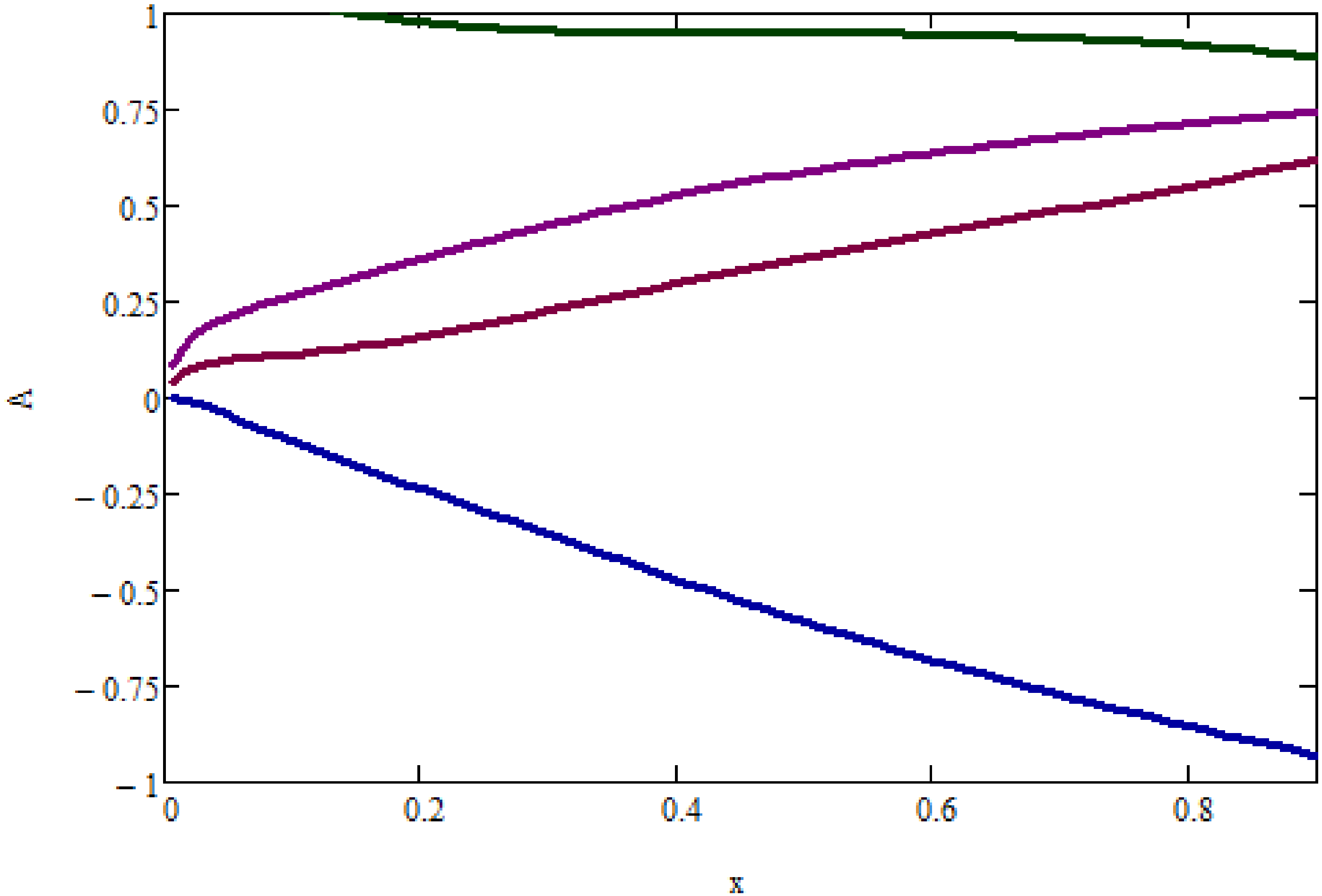

Figure 4. Asymmetry: $A^{\pi^+-\pi^-}_{+,p}$, $A^{\pi^+-\pi^-}_{\ell^- p}$, $A^{\pi^+-\pi^-}_{-,p}$, $A^{\pi^+-\pi^-}_{\ell^+ p}$ for $y=0$.

Electromagnetic corrections

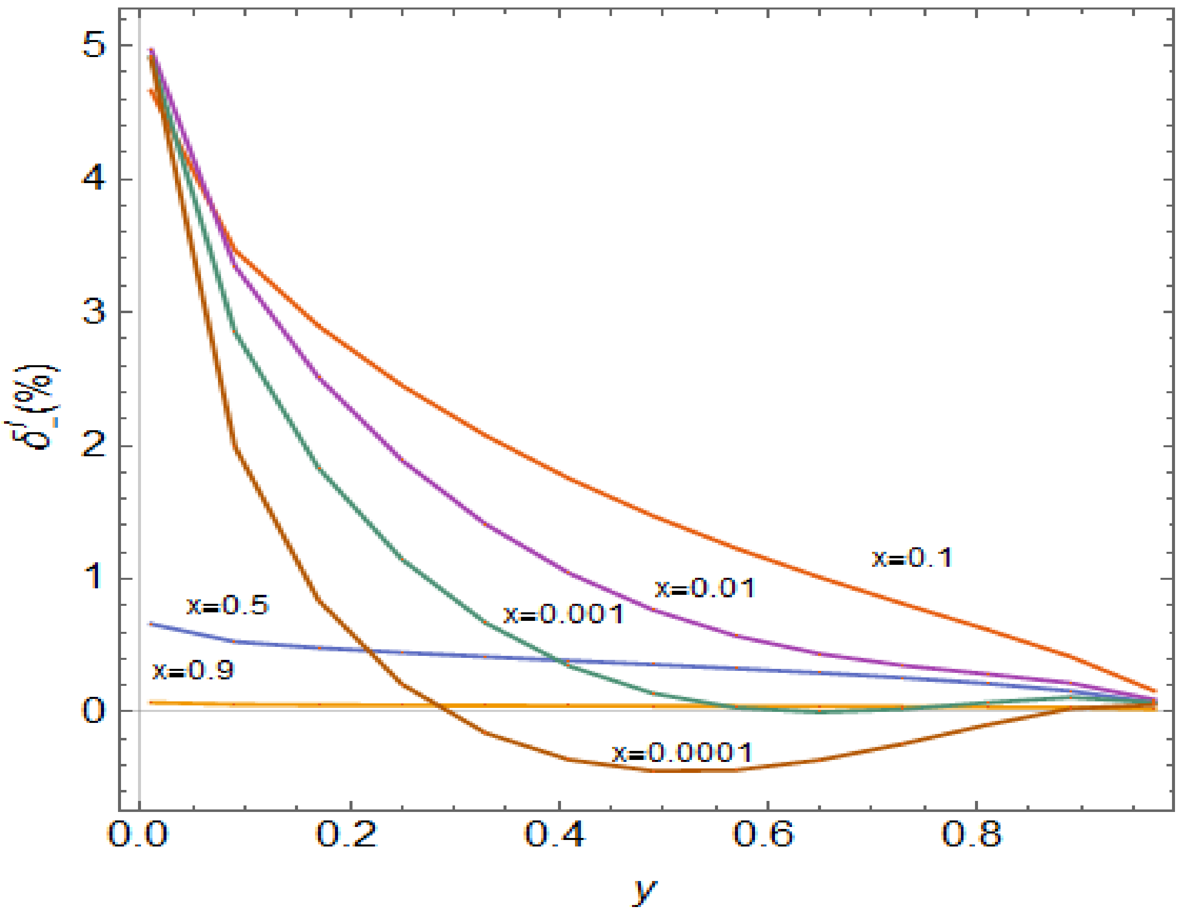

Figure 7. QED correction for asymmetry $A_{\ell^- p}^{\pi^+ - \pi^-}$ in LLA

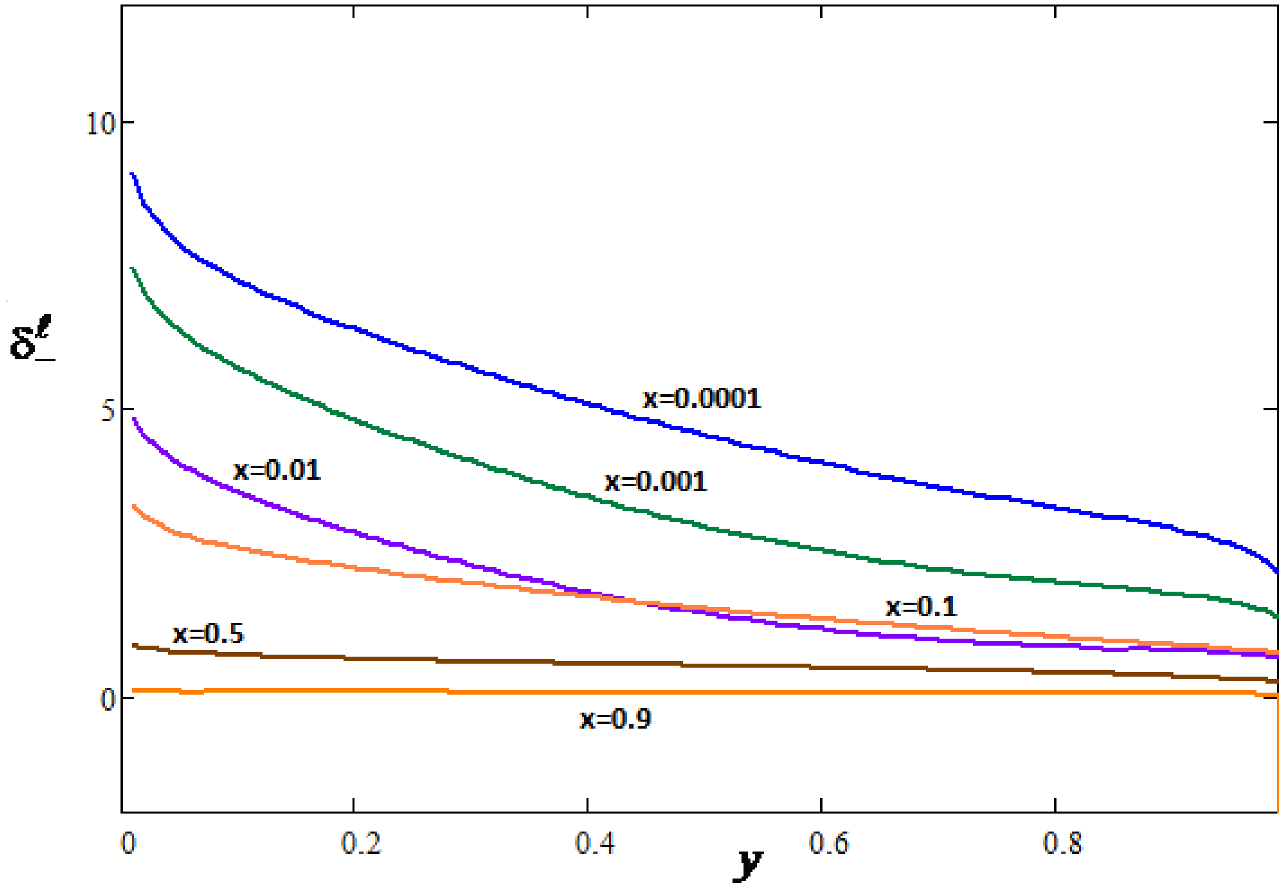

Figure 8. QED correction for asymmetry $A^{\pi^+ - \pi^-}_{\ell^- n}$ in LLA

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Conclusion

- Spin asymmetries are determined for the case of $\pi-$ meson production in the semi-inclusive $\ell N-$DIS. The quarks contribution to the nucleon spin are obtained through the measurable asymmetries.
- Numerical computations of asymmetries and electromagnetic correction are provided.