Nuclear Instruments and Methods in Physics Research B 355 (2015) 17–23

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

# Spontaneous and induced radiation by electrons/positrons in natural and photonic crystals. Volume free electron lasers (VFELs): From microwave and optical to X-ray range

V.G. Baryshevsky

*Research Institute for Nuclear Problems, Bobruiskaya str., 11, 220030 Minsk, Belarus*

ARTICLE INFO

*Article history:*
Received 17 November 2014
Received in revised form 1 April 2015
Accepted 1 April 2015
Available online 16 April 2015

*Keywords:*
Photonic crystal
Free electron laser
Induced radiation
Channeling radiation
Inverted free electron laser

ABSTRACT

Spontaneous and induced radiation produced by relativistic particles passing through natural and photonic crystals has enhanced capabilities for achieving the radiation sources operating in different wavelength ranges. Use of a non-one-dimensional distributed feedback, arising through Bragg diffraction in spatially periodic systems (natural and artificial (electromagnetic, photonic) crystals), establishes the foundation for the development of volume free electron lasers/masers (VFELs/VFEMs) as well as high-energy charged particle accelerators. The analysis of basic principles of VFEL theory demonstrates the promising potential of VFELs as the basis for the development of high-power microwave and optical sources.



## 1. Introduction

Recent advances in generation of electromagnetic radiation from relativistic particles led to the extension of the concepts and techniques used in this field throughout the investigations performed for different wavelengths from microwave to optical and X-ray. In particular, theoretical and experimental studies of spontaneous X-ray radiation in crystals (parametric X-ray radiation, diffracted radiation from relativistic channeled particles, diffracted radiation from a relativistic oscillator formed by a particle moving in the electromagnetic-wave field, and surface X-ray radiation from relativistic particles) gave rise to the idea of applying natural and artificial (photonic) crystals and diffraction gratings to form a two(three)-dimensional distributed feedback in free electron lasers (masers) [1–7]. This new generator type was called the volume free electron laser/maser (VFEL/VFEM). It was shown [2] that the law of radiative instability of relativistic beams passing through a crystal, discovered in [1] while studying the possibility of X-ray laser generation in crystals, can appreciably reduce the lasing threshold current in VFELs (VFEMs) as compared to conventional FELs (FEMs), backward wave oscillators (BWOs), and traveling wave tubes (TWTs). The two- or three-dimensional feedback used in VFEL (VFEM) resonators makes them similar to a system of several phase-locked generators. As a result, for microwave and optical ranges, VFELs (VFEMs) offer a benefit of significantly smaller size and higher radiation power over conventional FELs (FEMs), BWOs, and TWTs, which makes them attractive for use in heating thermonuclear plasma, long-distance power transmission, etc. Since in the case of multi-wave diffraction photonic crystals exhibit a high quality factor, they hold promise toward high-rate acceleration of relativistic particles in the inverse VFEL (VFEM) scheme.

## 2. Spontaneous emission of photons from relativistic particles in crystals

### 2.1. Resonant and parametric (quasi-Cherenkov) X-ray radiation

In 1946, Ginzburg and Frank showed [8] the existence of transition radiation when a relativistic particle moving at constant velocity passes through a matter-vacuum boundary. It was initially considered that the spectrum of transition radiation, like that of Cherenkov radiation, lies in the optical range [9]. But Garibyan [10] and Barsukov [11] demonstrated that as the particle energy increases, the spectrum of transition radiation shifts to the X-ray range, and the maximum frequency of radiation

$$\omega_{max} = \omega_L \gamma \quad (1)$$

*E-mail address:* bar@inp.bsu.by

http://dx.doi.org/10.1016/j.nimb.2015.04.004
0168-583X/

is proportional to the particle's Lorentz factor

$$\gamma = \frac{E}{mc^2} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}.$$

Here $m$ is the mass of the emitting particle, $\omega_L = \left(\frac{4\pi\rho e^2}{m_e}\right)^{1/2}$ is the Langmuir frequency of the medium, $\rho$ is the number of electrons per 1 $cm^3$, $m_e$ is the electron mass, and $e$ is the electron charge.

Naturally, the discovery of a new type of radiation in the X-ray range prompted a lot of research, particularly, an intense study of the features of transition radiation produced by a particle passing through periodically spaced plates (spatially periodic medium). Finally, Ter-Mikaelyan [12–14] showed that in this situation a new type of $\gamma$-radiation is produced and called it the resonant radiation of $\gamma$-quanta.

The frequency $\omega$ of resonant radiation increases as the particle energy is increased and (when the particle's velocity direction coincides with the $z$-axis along which the medium is periodic) can be written in the form:

$$\omega = \frac{\frac{2\pi n}{d} v}{1 - \frac{v}{c}\sqrt{\varepsilon_0}\cos\vartheta}, \tag{2}$$

where $d$ is the spatial period of the medium and $n$ is the integer. The dielectric permittivity of the medium $\varepsilon = \varepsilon_0 + \varepsilon_1(z)$, where $\varepsilon_1(z)$ is the periodic function of $z$ with a period $d$.

Now, let us consider resonant radiation from a viewpoint of an expert in microwave electronics (see, e.g., [15]). We shall immediately see that (1) describes the frequency of diffracted radiation emitted by a particle passing through a periodic medium (or in a vacuum along the periodic grating). In a microwave (optical) range, diffracted radiation is accompanied by Cherenkov radiation, because $\varepsilon_0 > 1$ for many media. At frequencies greater than the characteristic oscillation frequencies of electrons in atoms, the following universal expression [16] is true for the dielectric permittivity of a medium, and hence for the refractive index $n(\omega)$:

$$\varepsilon = 1 - \frac{\omega_L^2}{\omega^2}. \tag{3}$$

According to (3), the dielectric permittivity $\varepsilon < 1$ (the refractive index $n = \sqrt{\varepsilon} < 1$), and Vavilov–Cherenkov radiation should not exist in the X-ray range. However, Baryshevsky showed in 1971 [17] that the diffraction of photons emitted by a high-energy particle passing through a crystal can cause a phenomenon similar to induced (and hence spontaneous) X-ray Vavilov-Cherenkov radiation. Diffraction of the emitted photons also has an appreciable effect on the characteristics of transition radiation and bremsstrahlung (for details, see [6,7,17]).

The new type of radiation was called the parametric X-ray radiation (PXR). It appears because photons in periodic media, such as crystals, have several refractive indices $n$, some of which are greater than unity ($n > 1$) even in the X-ray range. In contrast to resonant (diffracted) radiation, the frequency of PXR photons emitted in the forward direction is independent of the particle energy and is completely determined by the properties of the crystal. In other words, PXR generation in crystals is accompanied by the excitation in the X-ray range of slow ($n > 1$) and fast ($n < 1$) waves. The existence of slow and fast waves in the microwave range is well known and widely used [18–20]. It is noteworthy that the presence of several refractive indices in crystals dictates replacing in (2) of the refractive index $n = \sqrt{\varepsilon_0}$, describing the spectrum of resonant radiation, by $n_i$, one of the crystal's refractive indices.

A significant research effort into the emission of X-ray photons from relativistic particles moving at constant velocity in crystals [6,7] followed the publication of [17,21]. In 1985, the phenomenon of PXR was experimentally observed [22,23]. The application potential of PXR has made it the subject of extensive study in many research centers worldwide [6,7,24–26].

Of course, the phenomena similar to quasi-Cherenkov parametric X-ray radiation in crystals, considered here, also occur when a particle moves in or near the surface of artificial three(two)-dimensional spatially periodic structures (electromagnetic and photonic crystals) [2]. In a particular case of particle motion in a one-dimensional spatially periodic medium, the phenomena accompanying quasi-Cherenkov generation of slow waves are thoroughly studied in a microwave range, where spontaneous radiation is difficult to observe, whereas induced radiation is easy to excite. This fact underlies the development of various types of generators: BWO, TWT, and others [19,20].

### *2.2. Channeling radiation of relativistic particles in crystals, diffracted channeling radiation, diffracted radiation of a relativistic oscillator*

Charged particles incident on a crystal at small angles with respect to crystallographic planes are captured into channeling regime. The limited character of particle motion in the transverse plane leads to particle oscillations in this plane. Such oscillations of charged particles (lying in the optical range of frequencies) cause the generation of X-ray and $\gamma$ radiation due to the Doppler effect. This phenomenon, predicted theoretically in 1976 by Baryshesky and Dubovskaya [27] and M. Kumakhov [28], was called the channeling radiation. A particle channeled in a crystal can be considered as a two(one)-dimensional relativistic oscillator. From a quantum–mechanical viewpoint, channeling radiation occurs through radiative transitions between the levels of transverse motion of particles passing through the crystal [7,27]. The first observations of channeling radiation were reported in [29–31], and a considerable amount of work has been published on this phenomenon since then.

The main characteristics of radiation produced by channeled particles can be obtained from the following simple reasoning [7,27,32–34]. Let a particle with the momentum $\vec{p}$ and the energy $E$ fall upon a plane-parallel crystal plate. Colliding with the crystal, it emits a photon of energy $\omega$ and momentum $\vec{k}$. In the final state, the particle energy and momentum take on the values $E_1$ and $\vec{p}_1$. It is important to remember that if the reaction proceeds in an arbitrary constant field, the energy (not the momentum) of the system is conserved. Thus, for particle energies we have the equality

$$E = E_1 + \omega. \tag{4}$$

Here we use $\hbar = c = 1$.

Due to the periodicity in the transverse plane of the crystal potential responsible for channeling, the transverse component of the momentum retains accurate to the reciprocal lattice vector $\vec{\tau}$ of the crystal

$$\vec{p}_\perp = \vec{p}_{1\perp} + \vec{k}_\perp + \vec{\tau}_\perp. \tag{5}$$

In the longitudinal direction, the potential responsible for channeling is constant, and the particle has a certain longitudinal momentum $p_{zn}$

$$p_{zn} = p_{1zn} + k_z n(k_z), \tag{6}$$

where $n(k_z)$ is the refractive index of the crystal (considered to be real). In writing (5) and (6) we took into account that at radiation in a finite crystal plate, the transverse component of the photon momentum does not change through refraction at the boundary, but its longitudinal component does.

Let us analyze (6), determining the change in the particle longitudinal momentum through photon emission. We shall write the explicit form of (6) in terms of particle energy:

$$p_{zn} = \sqrt{p^2 - 2m\varepsilon_{n\kappa}(E)}; \quad p_{1zf} = \sqrt{p_1^2 - 2m\varepsilon_{f\kappa_1}(E_1)}, \tag{7}$$

where $\varepsilon_{n\kappa}(\varepsilon_{f\kappa_1})$ are the energy levels of particle transverse motion, $\kappa$ is the quasi-momentum corresponding to the transverse momentum $\vec{p}_\perp$ of the particle in the initial state, and $\kappa_1$ is the quasi-momentum of the particle in the final state [27].

Let us consider the case of radiation when the frequency $\omega \ll E, E_1$. Under such conditions we can write [27]

$$\omega[1 - \beta n(\omega)\cos\vartheta] - \frac{m}{E}(\varepsilon_{n\kappa} - \varepsilon_{f\kappa_1}) = 0. \tag{8}$$

It follows from (8) that

$$\omega = \frac{(\varepsilon_{n\kappa} - \varepsilon_{f\kappa_1})\gamma^{-1}}{1 - \beta n(\omega)\cos\vartheta}. \tag{9}$$

To clarify the meaning of (9), let us compare it with the expression determining the frequency of photons emitted by the oscillator moving in a medium:

$$\omega = \frac{\Omega}{1 - \beta n(\omega)\cos\vartheta}, \tag{10}$$

where $\Omega$ is the oscillator frequency in the laboratory frame of reference; $\Omega = \Omega_0\sqrt{1-\beta^2} = \Omega_0\gamma^{-1}$; $\Omega_0$ is the oscillator frequency in its rest frame. Comparing (9) and (10), we can notice that a particle under channeling conditions may be considered as an oscillator moving in a medium and having the following frequency in its rest frame

$$\Omega_{0nf} = \varepsilon_{n\kappa} - \varepsilon_{f\kappa_1}. \tag{11}$$

Thus, the frequency $\Omega_{0nf}$ is determined by the difference of energies between the discrete zones (levels) of particle transverse motion [27].

In the laboratory frame, the frequency of such an oscillator is

$$\Omega_{nf} = (\varepsilon_{n\kappa} - \varepsilon_{f\kappa_1})\gamma^{-1} = \varepsilon'_{n\kappa} - \varepsilon'_{f\kappa_1}. \tag{12}$$

It should be pointed out that in contrast to conventional oscillators, the frequency of the oscillator corresponding to a channeled particle in the rest frame depends on the particle energy, because the value of the potential $V_c(\vec{\rho})$, produced by crystal axes (planes), depends on particle energy $V_c(\vec{\rho}) = \gamma V(\vec{\rho})$ ($V_c(\vec{\rho})$ is the potential of axes (planes) in the laboratory frame).

### 2.3. Diffracted X-ray radiation from a relativistic oscillator in a crystal (DRO) (diffracted channeling radiation (DCR))

X-ray quanta emitted by a channeled charged particle may undergo diffraction in a crystal. Depending on the frequency and the propagation direction of the X-ray photon under Bragg diffraction conditions, the photon has several refractive indices $n_i(\omega, \vec{k})$. As a result, under diffraction conditions the radiation of a relativistic oscillator is essentially modified as well. For example, radiation at large angles relative to the direction of particle motion becomes possible. The periodic structure of a crystal leads to modification of both particle motion and photon state. As a result, diffracted radiation of a relativistic oscillator (DRO) (diffracted channeling radiation) is formed. The radiation spectrum may be determined by analyzing the equation of the form

$$\omega(1 - \beta n_i(\omega, \vec{k})\cos\vartheta) - \Omega_{nf} = 0. \tag{13}$$

Moreover, the Vavilov–Cherenkov condition can be fulfilled, because under diffraction conditions at least one of the several crystal's refractive indices $n_i(\omega, \vec{k})$ may be greater than unity, i.e., the equation

$$1 - \beta n_i(\omega, \vec{k})\cos\vartheta = 0 \tag{14}$$

can hold true.

As a result, along with the radiation produced through radiative transitions between the discrete levels (zones) of transverse motion of a relativistic particle in a crystal, there exists parametric (quasi-Cherenkov) X-ray radiation (PXR) produced through longitudinal motion of particles in a crystal at constant velocity $\vec{v}$ determined by the particle longitudinal momentum. It should be noted [9] that the ordinary Cherenkov radiation can be considered as a specific case of radiation from the oscillator with zero eigenfrequency. Similarly, the PXR frequency in a periodic medium can be expressed by (13) (and hence, by (14)) in a specific case when $\Omega_{nf} = 0$. Thus the modification of refractive indices in the crystal under diffraction conditions leads to the appearance of two types of radiation (PXR and DRO) with angular distribution forming a diffraction pattern determined by the parameters of the crystal.

The considered features of radiation from a fast-moving oscillator in a crystal are of general character. They occur for relativistic particles and the associated oscillators moving in an artificial (photonic) crystal. Let us note in addition that similar phenomena also occur when a charged particle moves along the crystal surface or between two crystals in a vacuum [2]. In this case, surface quasi-Cherenkov (parametric) radiation arises (Fig. 1). With an electromagnetic wave (undulator) present in this region, surface diffracted radiation of a relativistic oscillator occurs.

A relativistic oscillator can be formed not only by an unperturbed crystal channel, but also by an external ultrasonic or laser wave which propagates in the crystal, creating a bent crystal channel (crystal undulator) [33–38]. According to [7,33], in the presence of an external variable field (for example, an ultrasonic field), a crystal is characterized by an effective index of refraction, depending on the external field parameters. By varying these parameters we can change the properties of parametric radiation and diffracted radiation of the oscillator. Diffracted radiation of the oscillator formed by an ultrasonic wave was considered in [39].

## 3. Photon radiation from a spatially modulated relativistic bunch of charged particles in natural and photonic crystals

Until now we have considered the process of emission of X-ray quanta by a single relativistic particle. It is well known, however, that a bunch of charged particles can emit far more intensely if its dimensions are much smaller than the radiation wavelength. Under such conditions, the intensity of radiation appears to be proportional to $N_b^2$ rather than to $N_b$ ($N_b$ is the number of particles in the bunch). In the case of microwave radiation, for example, having the wavelength $\lambda$ of several meters, it is quite possible to produce a relativistic electron bunch whose dimensions are smaller than $\lambda$. However, in the optical, especially in the X-ray, range a particle

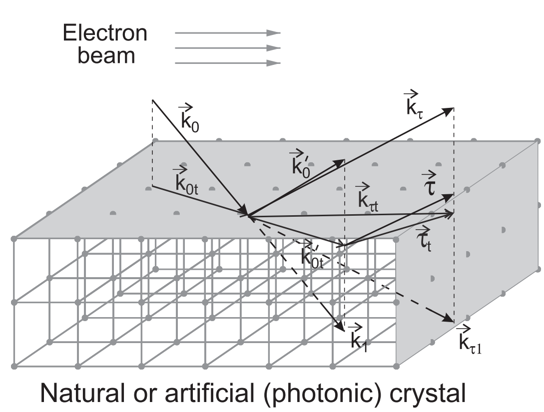


**Fig. 1.** Surface quasi-Cherenkov (parametric) radiation.

bunch with dimensions smaller than $\lambda$ is difficult to produce. The solution, nevertheless, was found after there came an understanding that the radiation power density of a spatially modulated beam can rise sharply against the radiation power density of an unmodulated beam.

In this regard, it would be useful to consider the possibility to apply spatially modulated beams with the period within the X-ray wavelength range for enhancing the radiation intensity of the electron beams traversing the crystal [4,5,40]. At present, such beams are produced in free electron lasers operating in the X-ray range (DESY and other X-ray FELs).

Let us introduce the beam density $\rho(\vec{r})$ and consider the beam modulated as $\rho(\vec{r}) = \rho_0 + \rho_1 \cos(\vec{\tau}\vec{r})$. As a result, the total number of quanta emitted by the bunch equals [4,5,40]

$$N = N_e\left(N_1 + c\frac{d^2N_1}{d\omega d\Omega}\bigg|_{\vec{K}=\vec{\tau}}\frac{\pi}{2}\mu^2\frac{\rho_0}{k_0^2}\right), \qquad (15)$$

where $\mu = \rho_1/\rho_0$; $N_1$ is the number of quanta of incoherent spontaneous radiation, which are produced by a single electron traversing the target under study; $k_0 = |\vec{\tau}| = \frac{2\pi}{\lambda_0}$; $\lambda_0$ is the radiation wavelength equal to the spatial modulation period $d$ of the beam; $c$ is the speed of light.

The ratio of the total number of coherently emitted quanta to that of incoherently emitted quanta is

$$\frac{N_{\rm coh}}{N_{\rm incoh}} \simeq \frac{\pi}{2}\frac{\mu^2\rho_0}{k_0^2\Delta k\Delta\Omega}. \qquad (16)$$

Here $\Delta k(\Delta\Omega)$ is the characteristic range $k$ (of solid angles), where the quantum is emitted through incoherent spontaneous radiation: $\Delta k/k \sim 1/\gamma$, $\Delta\Omega \sim 1/\gamma^2$, where $\gamma$ is the particle Lorentz factor. In obtaining (16) we used the estimate $N_1 \simeq \frac{d^2N_1}{dkd\Omega}\Delta k\Delta\Omega$. The ratio of the spectral densities of radiation within the range of the emission angles $\Delta\vartheta \sim 1/kL_{\perp b} \ll 1/\gamma$ of coherent radiation is $\mu^2 N_e$ ($L_{\perp b}$ is the transverse beam dimension).

Thus, within this range of emission angles, the intensity of coherent spontaneous radiation exceeds that of incoherent spontaneous radiation when the modulation depth is $\mu \geqslant 1/\sqrt{N_e}$. For example, when the number of electrons in the bunch $N_e \simeq 10^{12}$, it is sufficient that $\mu \geqslant 10^{-6}$, which enables one to appreciably simplify the problem of experimental observation of coherent radiation in the 10–100 keV X-ray range [4,5,40]. As follows from the above analysis, using a spatially modulated beam, we can observe coherent parametric X-ray radiation.

Transition radiation of a modulated beam was considered in [41]. K. A. Ispirian [42] gave a detailed study of the possibilities to use various mechanisms of spontaneous radiation for modulated beam diagnostics. Spontaneous superradiation from the particle bunch, leading to modulation of the initially unmodulated beam, was considered in [43,44].

## 4. Induced radiation in a three(two-) dimensional spatially periodic resonator (natural or artificial (photonic) crystal)

High spectral and angular densities of parametric (quasi-Cherenkov) and diffracted radiation of the oscillator (DRO) give a basis for using the considered mechanisms of spontaneous X-ray radiation to observe induced X-ray radiation. Generation of induced radiation in a crystal can be considered as generation in a crystal X-ray free electron laser (FEL) [4,5].

The main feature of the crystal X-ray FEL is that the crystal target not only forms the mechanism of spontaneous radiation, but also acts as a three-dimensional resonator for X-ray radiation, producing a distributed feedback (DFB). Because the energy of electrons is much greater than the energy of emitted photons, the methods developed for describing induced radiation in the microwave range are well applicable to study the phenomena occurring in crystal X-ray FELs. This enabled formulating the conditions of radiative instability of a beam of relativistic electrons (positrons) passing through a crystal [1–5]. Let us recall that the analysis of radiative instability of an electron beam requires the solution of the dispersion equation $D(\vec{k},\omega)$, which yields the relation $\vec{k}(\omega)$ or $\omega(\vec{k})$ [19]. Crystals in this case are considered as resonators in which a three(two)-dimensional distributed feedback is formed. As a result, the increment of radiative instability in the Compton regime was found to increase dramatically in the vicinity of the points of intersection of the dispersion equation roots. Particularly, if in the process of spontaneous emission two waves are excited in a crystal due to Bragg diffraction, then the increment of growth in the Compton regime appears to be proportional to $\rho^{1/4}$ ($\rho$ is the beam density) instead of $\rho^{1/3}$, as it occurs in FELs, BWOs, TWTs, etc. Under such conditions this law results in the reduction of the lasing threshold current $j_{th}$ as

$$j_{th} \sim \frac{1}{(kL)^{3+2s}},$$

where $s$ is the number of extra waves produced in the system through diffraction.

The law described here produced a significant effect on the very possibility of generating induced X-ray radiation. If we neglect this law, the generation threshold is achieved at inconveniently large values of the relativistic beam's current density: $j > 10^{13}$ A/cm$^2$. Under the conditions where several waves that appear through diffraction participate in the generation process, the generation threshold is reduced to $j \sim 10^8$ A/cm$^2$ for LiH crystals, which values are available at the facilities for X-ray FELs [4,5,34]. In fact, with increasing number of waves involved in the formation of a three(two)-dimensional distributed feedback, the quality factor (Q-factor) of the resonator grows. As a result, the lasing threshold current drops. Moreover, the considered features of generation of induced radiation in a three(two)-dimensional crystal (particularly, the reduction of the threshold current) are of general character and hold true for different types of spontaneous radiation in different ranges of wavelengths from X-ray to optical and microwave [2–5,7] (Fig. 2). This generality permits combining all generators using three(two)-dimensional distributed feedback under the one name: volume free electron lasers/masers (VFELs/VFEMs) [2–5].

The results [2–5] imply that the volume distributed feedback (non-one-dimensional feedback) has a number of advantages that make its application beneficial for generating stimulated radiation in a wide spectral range (with wavelengths from microwave and optical to angström). Moreover, generation in VFELs and VFEMs evolves in a large volume, and thus the electrical strength of the resonator is increased (the electromagnetic power and the electron beam are distributed over a greater volume). This feature of VFELs (VFEMs) is essential for generating power and super-power electromagnetic pulses. Mode discrimination in such oversized systems is carried out by multi-wave dynamical diffraction [4,5]. The energy transmitted by the electron beam to the radiation energy can be increased using VFELs (VFEMs) with variable parameters [45].

First lasing based on the above-described principles was experimentally obtained in mm-wavelength range in 2001 [3]. In the first experiment, the resonator of the generator was formed by two diffraction gratings with different periods: one for exciting the generation and the other for providing the feedback. In further experiments, the resonators were based on photonic crystals made

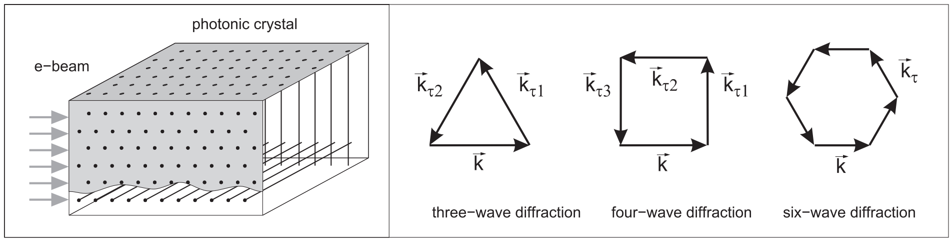


**Fig. 2.** Photonic crystal with the thread arrangement providing multi-wave volume distributed feedback. Threads are arranged to couple several waves (three, four, six, and so on), which appear due to diffraction in such structures in both vertical and horizontal planes. The electron beam (or several electron beams) passes through the photonic crystal.

from periodically strained threads. All these experiments supported the idea of using photonic crystals for generating radiation [4,5,46].

## 5. Use of a dynamical undulator mechanism to produce short-wavelength radiation in VFELs. Two-stage VFELs (VFEMs)

Low-relativistic electron beams in the undulator system can be used for radiating in a short-wavelength range, but in this case the undulator period must be small. For example, to obtain radiation with the wavelength from 0.3 mm to 1 mm at beam energy $E = 800$ keV–1 MeV, the undulator period must be $\sim$0.3–1 cm. The development of such undulators is a very complicated task. The use of a two-stage FEL with a dynamical wiggler generated by the electron beam [18] is a possible solution to this problem.

The above-described capability to significantly reduce the threshold currents and resonator dimensions enables the development of two-stage VFELs (VFEMs) with a dynamical wiggler generated by electron beams. A dynamical wiggler can be created with the help of any radiation mechanism: Cherenkov, Smith–Purcell, quasi-Cherenkov, or undulator. There is a possibility of smooth frequency tuning for both the pump and the signal waves by either varying the geometric parameters of the volume diffraction grating or by rotating the diffraction grating or the beam. The VFEL allows one to create a dynamical wiggler in a large volume, which is hard to achieve with a static wiggler. It should also be noted that in resonators with spatially-varied parameters of grating (photonic crystal), the electromagnetic wave with a spatial period depending on $z$ is formed. This means that a dynamical undulator with a period depending on $z$ appears along the whole resonator length, i.e., a tapering dynamical wiggler becomes settled, leading to an appreciable increase in the generation efficiency of FELs with such undulators (compare to FELs with conventional stationary undulators, described in [18]).

### *5.1. Acceleration in two(three)-dimensional spatially periodic structures. Mutual focusing of an electron beam and an electromagnetic wave in a Free electron laser. The possibility of optical phase conjugation in relativistic particle beams*

A high quality factor demonstrated by three(two)-dimensional electromagnetic (photonic) crystals in the case of multi-wave diffraction can be used to develop inverted VFELs, i.e., such crystals (resonators) can be used for particle acceleration [2,47], particularly for acceleration of particles from optical (X-ray) lasers in the range of super-high energies.

The motion of high-current electron beams in an electromagnetic undulator (static undulator or undulator produced by an electromagnetic wave) is accompanied by a variety of instabilities, which, as we showed earlier in this paper, change significantly if the electromagnetic wave and the particle beam move in a crystal. It was shown in [48] that parametric instability of a relativistic beam in the presence of a nonmonochromatic pump wave leads to a nonlinear contribution to the permittivity that is proportional to the local magnitude of $|\mathcal{E}_p(\vec{r},t)|^2$, where $\mathcal{E}_p(\vec{r},t)$ is the amplitude of the electric field strength of the pump wave at point $\vec{r}$ and time $t$. This allows an optical phase conjugation to occur in such beams. A detailed analysis was made by the example of the electron-beam interaction with a strong laser wave in a vacuum. It was shown that due to the interaction between the beam and the electromagnetic wave $\mathcal{E}_p$, the beam permittivity proves to be space periodic, the index of refraction of the beam turns out to be grater than unity as in the case of the FELs exploiting an undulator magnet [49–51]. Such a beam may be regarded as a waveguide for the electromagnetic wave. As a consequence, the photon and electron components attract each other and a bound electron-photon state is formed, where both the photon and the electron beams should be considered as dynamically bound subsystems [48,52,53]. In particular, under the conditions considered, the electromagnetic wave may also serve as a waveguide for the beam. As a result, the phenomenon of mutual focusing of the electron beam and light occurs [48,52,53]. According to the above-given analysis of radiative instability of a beam in a crystal, mutual focusing of the particle and laser beams is enhanced as the particle beam and the electromagnetic wave move in the crystal. Mutual focusing of the electron beam and the light wave enables the rotation of the light beam as the electron beam is rotated, and vice versa.

The experiments have now begun to accelerate electron bunches using laser radiation [54–56]. Due to high density of electron bunches and large amplitudes of laser field, the phenomena described in [48,52,53] can play an important part in further acceleration of the bunches and generation of radiation from them. The phenomenon of wave front reversal enables us to control the parameters of electromagnetic radiation and guide it to the target, which is important for practical applications, e.g. for heating thermal nuclear targets. As the particle moves in a bent light channel, not only bremsstrahlung appears, but also the beam's spin rotates, and the radiative self-polarization of electrons and positrons arises.

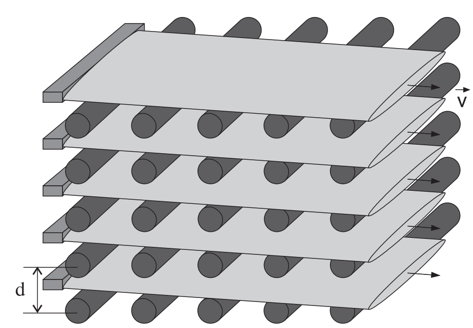


**Fig. 3.** VFEL (VFEM) with several beams.

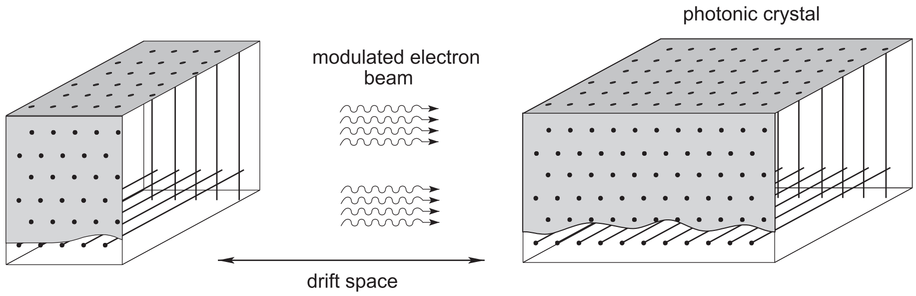


**Fig. 4.** Hybrid klystron-type VFEL (VFEM).

## 6. Volume free electron laser-self-phase-locking system

The possibilities of coherent combining of the fields generated by several microwave generators have been actively studied recently. Particularly, phase-locking is a subject of vigorous study [57–60]. The experiments confirm that, for example, using $N$ number of BWOs, one can develop relativistic microwave generators whose total power density will be as high as $W \sim N^2$ [59]. Two(three)-dimensional distributed feedback (DFB) arising in VFEL resonators with two-three dimensional spatially periodic structures (now often called electromagnetic or photonic crystals) enables producing coherent radiation from wide electron beams or several beams (see Fig. 3). A key feature of the VFEL is that as a result of diffraction, the signal is transferred from one point to another, thus linking the points of the beam and making them generate coherently (see Fig. 3). This means that the VFEL (VFEM) is a self-phase-locking generator of radiation (phase-locking inside). When $N$ number of electron beams pass through an electromagnetic crystal, the radiated power density increases in proportion to $N^2$ as a result of self-phase-locking of the beams due to two(-three)-dimensional DFB formed in the VFEL resonator. The theory of this phenomenon is given in [5,60].

## 7. Hybrid klystron-type VFELs (VFEMs)

One of the ways to increase the generation efficiency is the use of pre-modulated electron beams [19,20]. A remarkable embodiment of such FELs is the optical klystron [61]. Because the increment of radiative instability in VFELs (VFEMs) is large, we can use short sections formed by photonic crystals for beam pre-modulation (Fig. 4). The modulation is enhanced as the beam moves in the drift space, and the modulated beam enters into the second photonic crystal. For VFELs (VFEMs) and hence for construction of hybrid systems, some other mechanisms can be used for beam pre-modulation in microwave range, too [4,5]. Particularly, to pre-modulate the beam, a vircator [62–64] or a split-cavity oscillator (SCO) [65,66] can also be used. For a nonrelativistic case, the split-cavity oscillator was first suggested and studied in [65]; the SCO structures in the relativistic case were considered in [66]. What is more, according to [67–69], the application of metal inserts (meshes, grids, and so on) inside the resonator enables increasing the electron-beam-limiting current. Therefore, in generators having a photonic crystal as a resonator (grid traveling wave tube (TWT), VFEL, or VFEM), the presence of a metal grid (photonic crystal) serves both for forming the resonator, where interaction of the beam and the radiation occurs, and for increasing the beam vacuum limiting current.

## 8. Conclusions

Use of a non-one-dimensional distributed feedback in vacuum electronic devices removes the limits for available output power and promotes the development of highly stable mode-selective overmoded resonators. Such resonators open up new possibilities in the range where the wavelength is much less than the size of the resonator. Moreover, VFEL frequency can be tuned by rotating the resonator relative to the direction of the beam motion (or the beam relative to the resonator), and three(two)-dimensional distributed feedback in a multi-wave case allows a significant reduction of the lasing threshold. A high quality factor demonstrated by three(two)-dimensional electromagnetic (photonic) crystals (resonators) in the case of multi-wave diffraction can be used to develop inverted VFELs.

## References

[1] V.G. Baryshevsky, I.D. Feranchuk, Phys. Lett. A 102 (1984) 141–144.
[2] V.G. Baryshevsky, Dokl. Akad. Nauk SSSR 299 (1988) 1363–1366.
[3] V.G. Baryshevsky et al., Nucl. Instr. Meth. A 483 (2002) 21–23.
[4] V.G. Baryshevsky, 2011, Available from: <arXiv:1101.0783[physics.acc-ph]>.
[5] V.G. Baryshevsky, 2012, Available from: <arXiv:1211.4769[physics.optics]>.
[6] V.G. Baryshevsky, I.D. Feranchuk, A.P. Ulyanenkov, Parametric X-Ray Radiation in Crystals: Theory, Experiment and Applications, vol. 213, Springer-Verlag, Berlin Heidelberg, 2005.
[7] V.G. Baryshevsky, High-Energy Nuclear Optics of Polarized Media, World Scientific Publishing, Singapore, 2012.
[8] V.L. Ginzburg, I.M. Frank, Zh. Eksp. Teor. Fiz. 16 (1946) 15–28.
[9] L.M. Frank, Vavilov–Cherenkov Radiation. Theoretical Questions, Nauka, Moscow, 1988 (in Russian).
[10] G.M. Garibyan, Zh. Eksp. Teor. Fiz. 37 (1959) 527–533.
[11] K.A. Barsukov, Zh. Eksp. Teor. Fiz. 37 (1959) 1106; K.A. Barsukov, Sov. Phys. JETP 10 (1960) 787.
[12] M.L. Ter-Mikaelyan, Dokl. Akad. Nauk SSSR 134 (1960) 318; M.L. Ter-Mikaelyan, Sov. Phys. Dokl. 5 (1961) 1015.
[13] M.L. Ter-Mikaelyan, Nucl. Phys. 24 (1961) 43–61.
[14] M.L. Ter-Mikaelyan, High-Energy Electromagnetic Processes in Condensed Media, Wiley, New York, 1972.
[15] B.M. Bolotovskii, G.V. Voskresenskii, Sov. Phys. Uspekhi 9 (1) (1966) 73–96.
[16] L.D. Landau, E.M. Lifshitz, L.P. Pitaevskii, Electrodynamics of continuous media, in: L.D. Landau, E.M. Lifshitz (Eds.), Course of Theoretical Physics, vol. 8, Butterworth-Heinemann, 1984.
[17] V.G. Baryshevskii, Dokl. Akad. Nauk BSSR 15 (4) (1971) 306–308.
[18] T.C. Marshall, Free Electron Lasers, Macmillan Pub. Company, London, 1985.
[19] D.I. Trubetskov, A.E.Khramov, Lekcii po electronike dlya physikov, vol. 1–2, Moscow, Fizmatlit, 2003–2004 (in Russian).
[20] V.L. Granatstein, I. Alexeff (Eds.), High-Power Microwave Sources, Artech House, Boston, London, 1987.
[21] V.G. Baryshevskii, I.D. Feranchuk, Sov. Phys. JETP 34 (3) (1972) 502–504.
[22] Iu.N. Adishchev et al., JETP Lett. 41 (1985) 361–363.
[23] V.G. Baryshevsky et al., Phys. Lett. A 10 (1985) 477–479.
[24] K.-H. Brenzinger et al., Z. Phys. A 358 (1) (1997) 107–114.
[25] W. Lauth et al., Eur. Phys. J. A 28 (1) (2006) 185–195.
[26] P. Rullhusen, X. Artru, P. Dhez, Novel Radiation Sources Using Relativistic Electrons, from Infrared to X-Rays: From Infrared to X-Rays, Synchrotron Radiation Techniques and Applications, vol. 4, World Scientific, 1997.
[27] V.G. Baryshevsky, I.Ya. Dubovskaya, Dokl. Akad. Nauk SSSR 231 (6) (1976) 1335–1338.
[28] M.A. Kumakhov, Phys. Lett. A 57 (1) (1976) 17–18.
[29] R.L. Swent et al., Phys. Rev. Lett. 43 (23) (1979) 1723–1726.
[30] M.J. Alguard et al., Phys. Rev. Lett. 42 (17) (1979) 1148–1151.
[31] N. Cue et al., Phys. Lett. A 80 (1) (1980) 26–28.
[32] V.G. Baryshevskii, I.Ya. Dubovskaya, Phys. Status Solidi (b) 82 (1) (1977) 403–412.
[33] V.G. Baryshevsky, Channeling, Radiation and Reactions in Crystals at High Energy, Bel. State Univers, Minsk, 1982.
[34] V.G. Baryshevsky, I.Ya. Dubovskaya, Diffraction phenomena in spontaneous and stimulated radiation by relativistic particles in crystals (Review),

Technical Report Lawrence Berkeley Lab., CA, United States, LBL-31695, ESG Note-162, 1991, 119p, http://dx.doi.org/10.2172/5808050.
[35] V.G. Baryshevsky, A.O. Grubich, I.Ya. Dubovskaya, Phys. Lett. A 77 (1980) 61–64.
[36] V.G. Baryshevsky, A.O. Grubich, I.Ya. Dubovskaya, Dokl. Akad. Nauk BSSR 24 (3) (1980) 226–229.
[37] V.V. Kaplin, S.V. Plotnikov, S.A. Vorobiev, Zh. Tekh. Fiz. 50 (1980) 1079–1081.
[38] V.G. Baryshevsky, V.V. Tikhomirov, Nucl. Instr. Meth. B 309 (2013) 30–36.
[39] V.G. Baryshevsky, I.Ya. Dubovskaya, J. Phys.: Condens. Matter 3 (14) (1991) 2421–2430.
[40] V.G. Baryshevsky, Vesti Akad. Nauk BSSR, Ser. fiz-math nauk 1 (1992) 31–37.
[41] N.A. Korkhmazian, L.A. Gevorgian, M.L. Petrosian, Zh. Tekh. Fiz. 47 (1977) 1583–1597.
[42] K.A. Ispirian, Nucl. Instr. Meth. B 309 (2012) 4–9.
[43] S.V. Anishchenko, V.G. Baryshevsky, 2013, Available from: <arXiv:1304.0960[physics.acc-ph]>.
[44] S.V. Anishchenko, V.G. Baryshevsky, 2014, Available from: <arXiv:1411.2960[physics.optics]>.
[45] V.G. Baryshevsky, A.A. Gurinovich, 2006, Available from: <arXiv:physics0608068[physics.acc-ph]>.
[46] V.G. Baryshevsky et al., in: Proceedings of the 35th Int. Conf. on Infrared, Millimeter, and Terahertz Waves (IRMMW-THz 2010), 5–10 September 2010, Rome, Italy, pp. 1–2 http://dx.doi.org/10.1109/ICIMW.2010.5612862.
[47] V.G. Baryshevsky, K.G. Batrakov, I.Ya. Dubovskaya, Vesti Akad. Nauk BSSR, ser. fiz.-energ. 1 (1991) 53.
[48] V.G. Baryshevsky, Phys. Lett. A 132 (1) (1988) 30–32.
[49] E.T. Scharlemann, A.M. Sessier, Phys. Rev. Lett. 54 (1985) 1925–1928.
[50] P. Sprangle, C.M. Tang, Appl. Phys. Lett. 39 (1981) 677–680.
[51] P. Sprangle, A. Ting, C.M. Tang, Phys. Rev. A 36 (1987) 2773–2781.
[52] V.G. Baryshevsky, I.Ya. Dubovskaya, O.N. Metelitsa, Pis'ma ZhTF 15 (2) (1989) 1–3.
[53] V.G. Baryshevsky, I.Ya. Dubovskaya, O.N. Metelitsa, Phys. Lett. A 148 (5) (1990) 272–274.
[54] X. Artru, K.A. Ispirian, M.K. Ispiryan, Available from: <arXiv:0707.0148v2[physics.acc-ph]>.
[55] S.B. Dabagov, A.V. Dik, E.N. Frolov, Available from: <arXiv:1404.3307v2[physics.acc-ph]>.
[56] Young-Min Shin, D.A. Still, V. Shiltsev, Phys. Plasmas 20 (2013) 123106, http://dx.doi.org/10.1063/1.4846760.
[57] C.D. Taylor, D.V. Giri, High Power Microwave Systems and Effects, Taylor and Francis Group, 1994.
[58] W. Woo et al., J. Appl. Phys. 65 (2) (1989) (0021-8979/89/020861-06).
[59] A.A. Elchaninov et al., Zh. Tekh. Fiziki 81 (1) (2011) 125–130.
[60] V.G. Baryshevsky, A.A Gurinovich, Available from: <arXiv:1301.4330[physics.acc-ph]>.
[61] N.A. Vinokurov et al., Rev. Sci. Instrum. 60 (7) (1989) 1435–1438, http://dx.doi.org/10.1063/1.1141084.
[62] L.S. Bogdankevich, A.A. Rukhadze, Sov. Phys. Uspekhi 14 (2) (1971) 163–189.
[63] A.E. Dubinov, V.D. Selemir, Radiotekhnika i electronika 47 (2002) 645.
[64] V.D. Selemir et al., Tech. Phys. Lett. 27 (7) (2001) 583–585.
[65] B.M. Marder et al., IEEE Trans. Plasma Sci. 20 (3) (1992) 312–331.
[66] V.G. Baryshevsky, Available from: <arXiv:1402.3403[physics.acc-ph]>.
[67] D.D. Ryutov, Pis'ma Zh. Tekh. Fiziki 1 (1975) 581.
[68] R.A. Richardson et al., J. Appl. Phys. 69 (1991) 6261–6272.
[69] R.J. Adler, B. Sabol, G.F. Kiuttu, IEEE Trans. Nucl. Sci. NS-30 (4) (1983) 3198–3200.