Nonlinear Phenomena in Complex Systems, vol. 25, no. 4 (2022), pp. 359 - 367

Nonlinear Dynamics of Radiation in Multiple-Beam Vacuum Electronic Devices

S. N. Sytova*
Institute for Nuclear Problems, Belarusian State University,
11 Bobruiskaya Str, 220006 Minsk, BELARUS
(Received 10 November, 2022)

The article is devoted to overview of different types of vacuum electronic devices with two or more charged particle beams. There are travelling wave and backward wave tubes, free electron lasers and masers, volume free electron lasers. Two different cases take place in such situation: multiple-beam instability in such devices and multiple-stream instability. In the first case some charged particle beams moves in the system with different velocities. In the second one there are beams with almost equal velocities (streams). Two systems of equations for volume free electron laser with two electron beams are proposed. Some numerical results of VFEL numerical simulation are given and discussed.

PACS numbers: 41.60.Cr; 35Q60, 35F20, 65M06
Keywords: simulation, nonlinear dynamics, multiple-beam instability, multiple-stream instability, vacuum electronic device, volume free electron laser
DOI: https://doi.org/10.33581/1561-4085-2022-25-4-359-367

1. Introduction

From the middle of the twentieth century to the present day, we observe a rapid development of research in the field of nonlinear processes arising during electromagnetic radiation by charged particles passing various types of electronic vacuum amplifiers and generators, operating in a wide spectrum range from X-ray to Terahertz. Over the past 80 years, thousands of articles, monographs and reviews have been devoted to various aspects of functioning of such vacuum electronic devices (VED), including the demonstration of the diversity and their obvious commonality in physical principles, as well as the complex nonlinear dynamics of their work. It concerns the following devices: travelling wave tubes (TWT), backward wave tubes and oscillators (named BWT and BWO, as well as karsinotrons and karsinotrodes), multi-wave Cherenkov generators, free electron lasers (FEL), gyrotrons and free electron masers (FEM), orotrons, various types of Cherenkov and Smith-Purcell FELs, volume free electron lasers (VFEL). A brief description of VED physical phenomena was given recently [1], [2]. References there contain both some of the earliest "classical"works in this field and some of the recent ones devoted to the theory, experimental research, mathematical modelling of VED as well as their chaotic dynamical nature.

From the very beginning of the VED development it was proposed to use two-stream, many-stream and multiple-beam instabilities to improve device performance.

The goal of the paper is devoted to overview different types of VED with two or more charged particle beams paying special attention to VFEL.

2. Multiple-beam vacuum electronic devices

The phenomenon of two-stream instability is a classic example of the instability in distributed conservative systems [3]. Moreover, this instability is a textbook example of convective and absolute instabilities. The two-beam instability has been known for a long time

*E-mail: sytova@inp.bsu.by

too. This term was first introduced in the study of the interaction of two electron beams, and later became widespread in various fields of science.

To eliminate misunderstandings, in this paper we consider that in the multiple-beam instability some charged particle beams moves in the system with different velocities. And in the multiple-stream instability there are beams with almost equal velocities (streams).

After creation of TWT by Rudolf Kompfner [4] in the mid-1940s, the era of devices with a long interaction section was opened. Various scientific groups have sought to improve their output characteristics. Attempts were also made to create more powerful devices, but in this case there were difficulties associated with overheating and failure of the slow-wave system (SWS) or resonator that are the "body" of devices. As a result, it was proposed to introduce a second electron beam, copropagating through SWS. It was a particularly good decision during the transition to the short-wave part of the microwave range with the possibility to increase the output power.

So, use of multiple beams, propagating along its own individual transit path through SWS, permits to obtain in the system higher total charge at less charge from each injector. This lead to lower emittance in the system, possible shorter wigglers and SWS and higher saturated powers.

The first papers devoted to such instability in different types of VED appeared at the same time as cited work by R.Kompfner [5]–[8], including klystrons [9]. A wide range of developed different multiple-beam vacuum electronic tubes (klystrons, TWT and BWO) was depicted in [10], [11] later.

First proposals to use some electron beams in FEL are dated by 1981-1982 [12], [13]. Works on multiple-beam FEL and FEM published 10 and 20 years later should be mentioned [15]–[17], as well as the recent works [18]–[21].

Nowadays the interest in the phenomenon of multiple-beam instability has reappeared in microwave electronics [22]–[25].

To develop multiple-beam VED, a special technique for generating multiple electron beams with different energies and comparable currents from a single cathode stalk was developed [26]. Some special proposals of VED are based on periodic slow wave structure (SWS) made from metamaterials [27] or in a sandwich structure composed of a bi-grating and a sub-wavelength holes array [28]. In each of these works an increase in the interaction power of electron beams was demonstrated. A new operating mode of the electron wave tube, which is called the interference mode, is described [29].

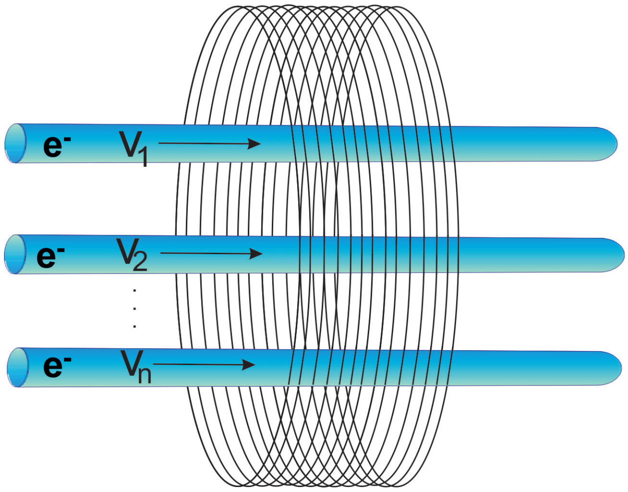

FIG. 1. Scheme of n beams moving through a spiral SWS.

To obtain the numerical results of simulation of considered VED in the recent studies, the fully electromagnetic, relativistic particle-in-cell codes MAGIC [30] and CST [31]were used.

Let us give a sketch describing the multiple-beam instability in VED and FEL (see Figs. 1–3). Here SWS gratings can be different comb, spiral, pin-type or slotted systems. The coupling between electromagnetic waves in different channels of SWS is carried out through the holes in the grating.

3. Volume free electron lasers

So, a close relative of considered electronic tubes and FEL is volume free electron lasers

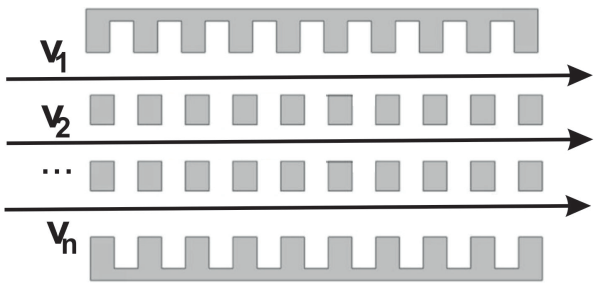

FIG. 2. Side view of n beams moving through SWS grating.

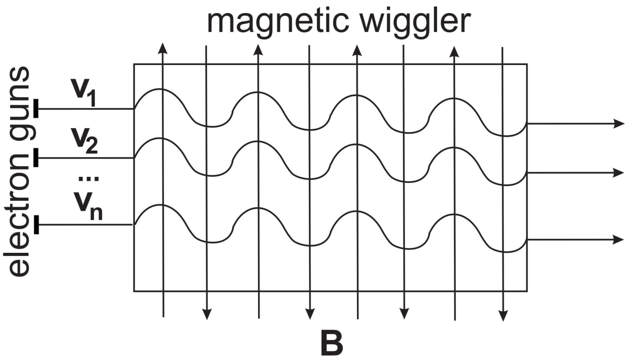

FIG. 3: Scheme of multiple-beam FEL.

[32], [33]. It works on the radiation of relativistic electron beam moving in two- and three-dimensional SWS (spatially-periodic structures or resonator) in synchronism with one or more coupled strong electromagnetic waves. For these waves the conditions of Bragg diffraction should be valid. Dynamical diffraction provides volume (non-one-dimensional) distributed feedback (VDFB) in contrast to one-dimensional distributed feedback in TWT, BWT etc.

VFEL SWS are natural crystals (in X-ray range [35]) or artificial electromagnetic (photonic) crystals with a period proportional to the radiation wavelength [33], [34]. It is shown [33] that VFEL physical principles are valid for all frequency ranges from X-ray to Terahertz.

In [36] a system of equations for two-wave VFEL simulation was proposed. Here the electron beam dynamics was presented in term of electron phase by averaging over the phases of electron entry into the interaction region with the respect to the moment of electron entrance and its transverse coordinate.

In [34] a system of equation, based on [36], was proposed for synchronism of one electron beam with two coupled strong electromagnetic waves. In [33] an idea of multiple-beam VFEL was expressed as laser-self-phase-locking system. Such systems consisting from some VEDs connected by bridge waveguides for their mutual synchronization were proposed and investigated theoretically, numerically and experimentally in [37]–[40]. In VFEL such "bridge waveguide"is the volume resonator, providing VDFB by dynamical diffraction.

In [41] a generalized system for simulation of multi-section multi-wave VFEL with some electron beams in very general outline including external mirrors is proposed. Also in [41] this scheme was simplified for the case of two-wave two-beam VFEL in Bragg geometry. We will refine this system below and formulate a system of equations for two-stream VFEL.

A two-beam two-wave VFEL sketch in Bragg geometry is given in Fig.4. Here two electron beams with electron velocity $\mathbf{u}_1$ and $\mathbf{u}_2$ pass through a spatially periodic resonator. Here two strong coupled electromagnetic waves under diffraction conditions are excited in the resonator. If simultaneously electrons of the first and the second beams are under synchronism condition each with their own wave, they emit electromagnetic radiation in directions depending on these diffraction conditions.

We do not discuss technical details and possible design of such VFEL here. Obviously, taking into account the modern achievements of vacuum electronics, the development of a structure and the passage of two beams through it in different directions are quite realistic.

Let us consider two additional schemes of two-beam two-wave VFEL in Bragg (Fig. 5) and Laue (Fig. 6) geometries. In Fig. 7 a two-stream VFEL in Laue geometry with two almost identical electron beams that are in synchronism with one of two electromagnetic waves is proposed. In all these figures vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ are the

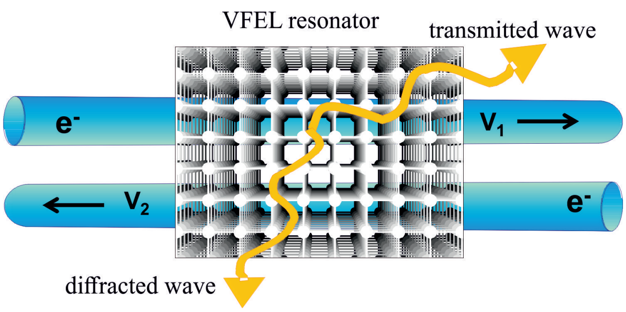

FIG. 4. Two-beam two-wave VFEL in Bragg geometry.

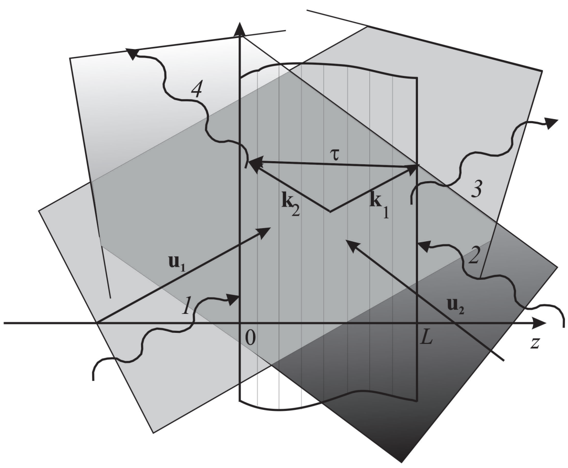

FIG. 5. Two-beam two-wave VFEL scheme in Bragg geometry.

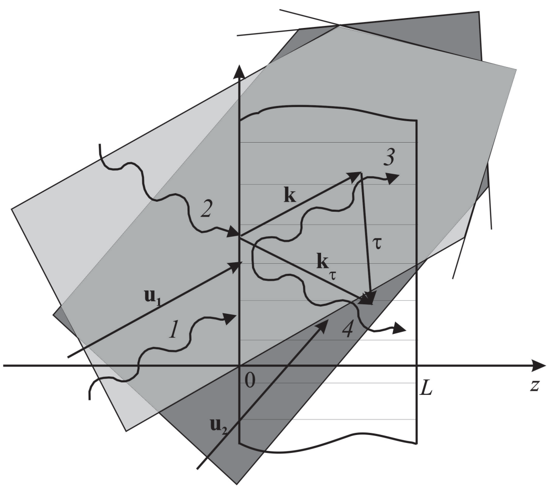

FIG. 6. Two-beam two-wave VFEL in Laue geometry.

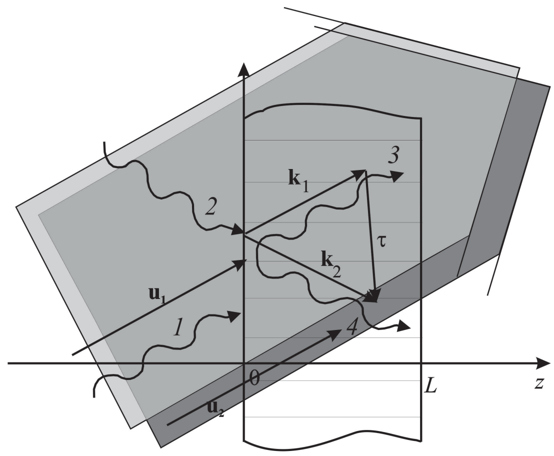

FIG. 7. Two-stream two-wave VFEL scheme in Laue geometry.

wave vectors of transmitted and diffracted wave, respectively.

The diffraction condition is expressed by the formula: $2\mathbf{k}_1\tau + \tau^2 \approx 0$ and $\mathbf{k}_2 = \mathbf{k}_1 + \tau$, τ is the SWS reciprocal lattice vector.

External electromagnetic waves marked by *1* and *2* with amplitude E_0^1 and E_0^2 and wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively, can fall on the system from outside. Electromagnetic waves marked by *3* and *4* with amplitude E_1 and E_2 and wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ under diffraction conditions are generated or amplified in the system if parameters of electron beams with velocities $\mathbf{u}_i$, beam densities n_i^b and current densities $\mathbf{j}_i = en_i^b\mathbf{u}_i, i = 1, 2$ exceed threshold values.

Synchronism conditions are the next:

$$\frac{|\omega - \mathbf{k}_i\mathbf{u}_i|}{\omega} \sim \delta_i \ll 1, i = 1, 2,$$

where δ_i are detuning from the Cherenkov condition for both beams.

Figs.5–7 as well as Fig.4 demonstrates the volume nature of distributed feedback and the main VFEL physical property, where the vectors of all electromagnetic waves and the vectors of velocity of particle beams are directed at angles with respect to each other.

4. System of VFEL equations

Based on [36] and [41], the system for two-beam two-wave VFEL has the following form:

$$\frac{\partial E_1}{\partial t} + \gamma_1^0 c \frac{\partial E_1}{\partial z} + 0.5 i\omega l_1 E - 0.5 i\omega \chi_\tau E_2 = I_1,$$

$$\frac{\partial E_2}{\partial t} + \gamma_2^0 c \frac{\partial E_2}{\partial z} - 0.5 i\omega \chi_{-\tau} E + 0.5 i\omega l_2 E_2 = I_2,$$

$$I_i = \Upsilon_i \int_0^{2\pi} (2\pi - p)(e^{-i\Theta_i(t,z,p)} + e^{-i\Theta_i(t,z,-p)})dp,$$

$$\frac{\partial^2 \Theta_i(t,z,p)}{\partial z^2} = \frac{e\Phi_i}{m\gamma_i^3 \omega^2}\left(k_{iz} - \frac{\partial \Theta_i(t,z,p)}{\partial z}\right)^3 \times \mathrm{Re}\left(E_i(t - z/u_i, z)e^{i\Theta_i(t,z,p)}\right),$$

$$E_1(t,0) = E_0^1, E_2(t,z_0) = E_0^2, E_i(0,z) = 0,$$

$$\Theta_i(t,0,p) = p, \quad \frac{\partial \Theta_i(t,0,p)}{\partial z} = k_{iz} - \omega/u_i,$$

$i = 1,2,\ t > 0,\ z \in [0,L],\ p \in [-2\pi, 2\pi],\ z_0 = 0$ in Laue geometry, $z_0 = L$ in Bragg geometry.

In this system of equations γ_i^0 are diffraction cosines. System parameters are the following: $l_i = (k_i^2 c^2 - \omega^2 \varepsilon_0)/\omega^2 + \delta_i$. γ_i are Lorenz-factors for corresponding beam velocities $\mathbf{u}_i$. $\Upsilon_i = e n_i^b u_i \Phi_i/(4\pi)$, $\Phi_i = \sqrt{l_i + \chi_0 - c^2/(u_i\gamma_i)^2}$. $\varepsilon_0 = 1 + \chi_0$ is a mean dielectric susceptibility , χ_0 $\chi_{\pm\tau}$ are Fourier components of the dielectric susceptibility of the resonator.

For two-stream two-wave VFEL in Laue geometry (Fig.7) we have the following system of equations:

$$\frac{\partial E_1}{\partial t} + \gamma_1^0 c \frac{\partial E_1}{\partial z} + 0.5 i\omega l_1 E - 0.5 i\omega \chi_\tau E_2 = I_1 + I_2,$$

$$\frac{\partial E_2}{\partial t} + \gamma_2^0 c \frac{\partial E_2}{\partial z} - 0.5 i\omega \chi_{-\tau} E + 0.5 i\omega l_2 E_2 = 0,$$

$$I_i = \Upsilon_i \int_0^{2\pi} (2\pi - p)(e^{-i\Theta_i(t,z,p)} + e^{-i\Theta_i(t,z,-p)})dp,$$

$$\frac{\partial^2 \Theta_i(t,z,p)}{\partial z^2} = \frac{e\Phi_i}{m\gamma_i^3 \omega^2}\left(k_{1z} - \frac{\partial \Theta_i(t,z,p)}{\partial z}\right)^3 \times \mathrm{Re}\left(E_1(t - z/u_i, z)e^{i\Theta_i(t,z,p)}\right),$$

$$E_i(t,0) = E_0^i, E_i(0,z) = 0,$$

$$\Theta_i(t,0,p) = p, \quad \frac{\partial \Theta_i(t,0,p)}{\partial z} = k_{1z} - \omega/u_i,$$

$i = 1,2,\ t > 0,\ z \in [0,L],\ p \in [-2\pi, 2\pi]$.

5. Numerical results

It has been shown [36] that VFEL is a chaotic system, namely multiple-beam multiple-wave VFEL [41]. The study of VFEL as a chaotic dynamical system is important to control chaos and to choose optimal set of VFEL parameters.

The VFEL nonlinear generation dynamics is due to the nonlocal nature of the interaction of electron beams with electromagnetic field under diffraction conditions and VDFB. When studying the VFEL chaotic nature, its space-time and phase dynamics were investigated, various dynamic modes of operation with complex transformation were obtained numerically [42].

Fig. 8 shows a parametric map of the transition to chaos in a two-beam two-wave VFEL for transmitted wave. Along the edges of the map the most typical dynamic chaotic regimes are shown. 0 describes here the area below the generation threshold. P demonstrates different periodic regimes, C presents chaos. Here on the graphs, the lower lines denote the transmitted wave, the upper lines are the diffracted wave.

Let us note, that in the absence of the second beam the threshold current density in the system is the next: $j_1 = 300$ A/cm^2. In the absence of

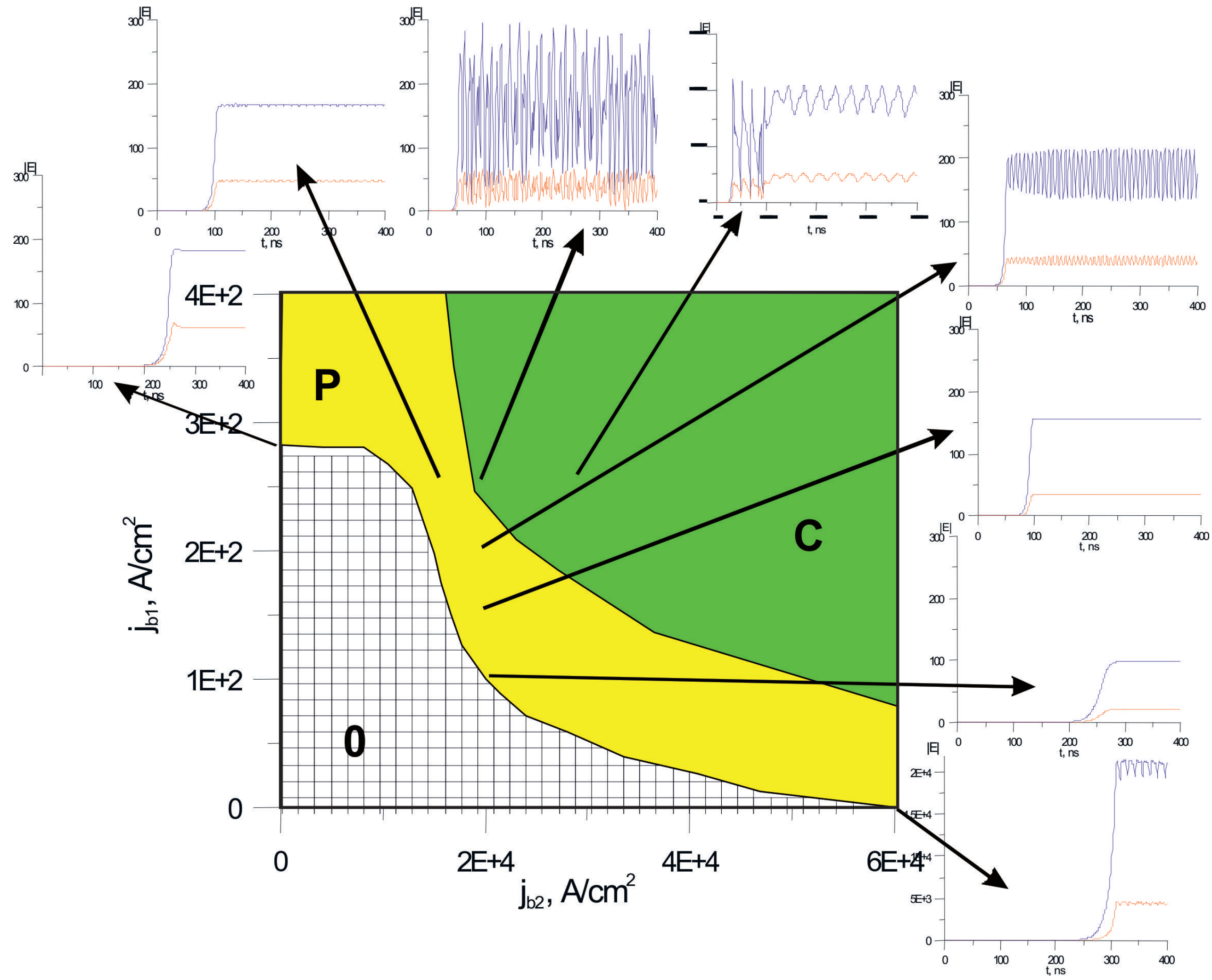

FIG. 8. (color online) Parametric map of the transition to chaos in a two-beam two-wave VFEL in Bragg geometry.

the first one the threshold current density of the second beam is j_2=6 kA/cm^2. So, the two-beam configuration allows to decrease considerably the current thresholds.

In Fig. 8 one can see that small changing of the first beam current density leads to considerable changing in dynamical regimes in the system from steady state to chaotic one including a periodicity band along the threshold line.

Essential changing in chaotic regimes can be seen in Figs. 9–**??** where the current density of the second beam j_2 varies at the constant value j_1=400 A/cm^2. This corresponds to the data on the top side of the square box in Fig. 8.

In total, all regimes in Figs. 9–**??** are chaotic but with changing from weak chaos to developed chaos and then through intermittency to almost periodic (weak chaos) behaviour.

6. Conclusion

An overview of different multiple-beam vacuum electronic devices is given including volume free electron laser. The performed computer simulation of two-beam two-wave VFEL demonstrates significant changes in the nature of the dynamic solution with respect to

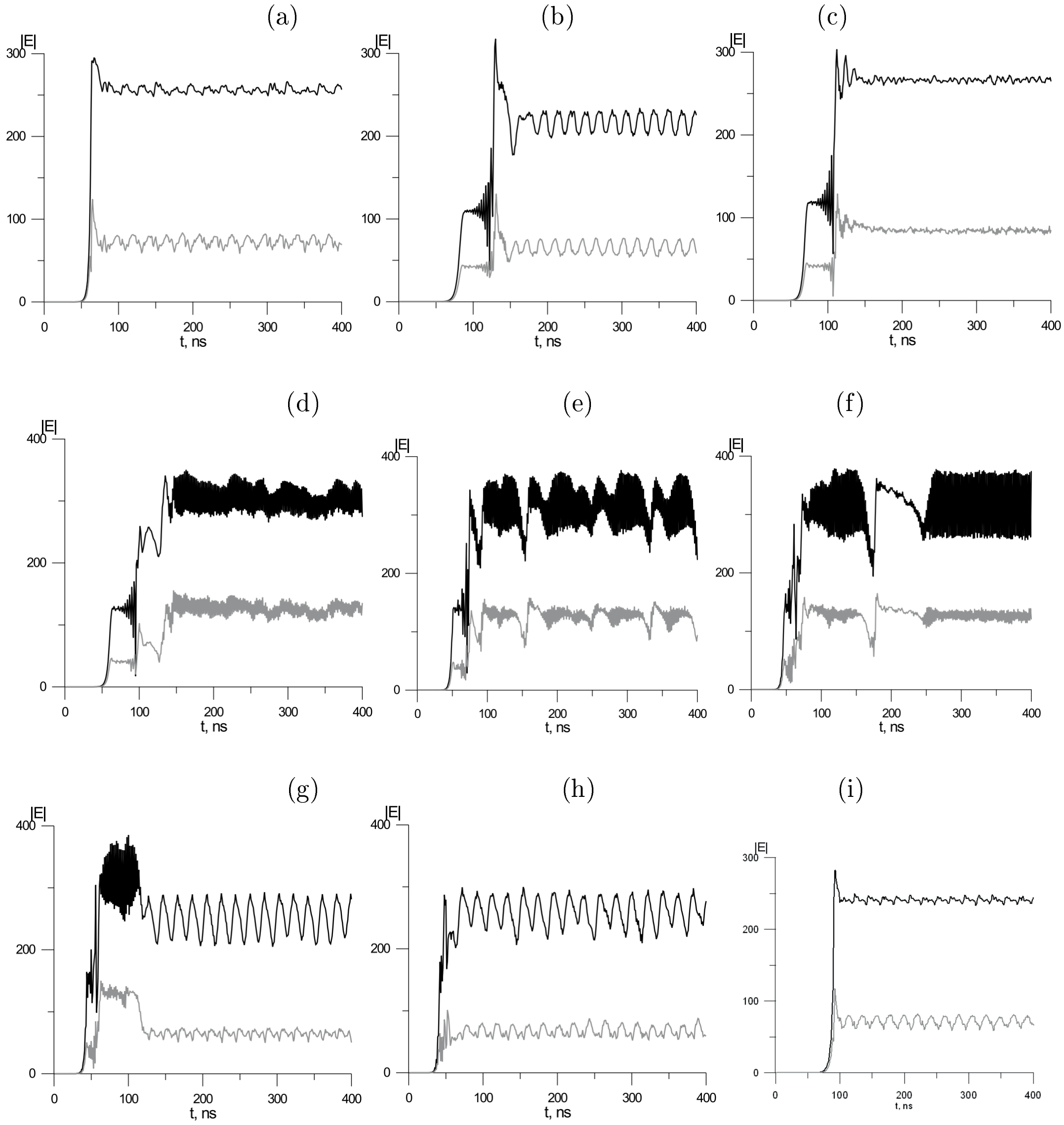

FIG. 9. Amplitudes of transmitted (grey lines) and diffracted (black lines) waves for different values of j_2; $j_2 = 0$ (a), 1.1 (b), 1.2 (c), 1.3 (d), 1.5 (e), 1.6 k (f), 1.7 (g), 1.8 k(h), 3 (i) kA/cm^2.

one-beam VFEL as well as in the generation thresholds. So, using of an additional beam in the system is a method of VFEL chaos dynamic control.

References

[1] S.N. Sytova. Nonlinear dynamics of radiation of high-current beams of charged particles in spatially periodic structures. Journal of the Belarusian State University. Physics. No.1, 62 (2021).

[2] S.N. Sytova. Simulation Techniques for Vacuum

Electronic Devices. Nonlin. Phen. Compl. Syst. **25**(2), 168 (2022).
[3] R. Briggs. *Electron stream interaction with plasmas.* (Cambridge, MIT Press, 1964).
[4] R. Kompfner. The traveling-wave tube as amplifier at microwaves. Proc. IRE. **35**(2), 124 (1947).
[5] L.S. Nergaard L.S. Analysis of a simple model of a two-beam growing-wave tube. RCA Rev. **9**, 585 (1948).
[6] A.V. Haeff. The Electron-Wave Tube-A Novel Method of Generation and Amplification of Microwave Energy. Proc. IRE. **37**, 4 (1949).
[7] A.V. Haeff. Discussion on "The Electron Wave Tube". Proc. IRE. **37**, 777 (1949).
[8] J.R. Pierce, L.R. Walker. Growing Waves Due to Transverse Velocities. The Bell System Technical Journal. 109 (1956).
[9] U.R. Boyd et al. The multiple-beam klystron. IRE Trans. Electron Devices. **EO-9**, 247 (1962).
[10] P.M. Phillips et al. Review of Two Stream Amplifier Performance. NRL Memorandum Report 6382, 30 p. (1989).
[11] A.S. Pobedonostev et al. Multiple-beam microwave tubes. 1993 IEEE MTT-S Inte. Microwave Symposium Digest. **2**, 1131 (1993).
[12] M.A. Piestrup. Multiple beam free-electron lasers and optical klystrons. Appl. Phys. Let. **39**, 696 (1981).
[13] G. Bekefi, K.D. Jacobs. Two-stream, free-electron lasers. J. Appl. Phys. **53**, 4113 (1982).
[14] G. Bekefi. Doublestream cyclotron maser. J. Appl. Phys. **71**, 4128 (1992).
[15] H. Wilhelmsson. Double Beam Free Electron Laser. Physica Scripta. **44**, 603 (1991).
[16] H.P. Freund, D. Douglas, P.G. Shea. Multiple-beam free-electron lasers. Nucl. Instr. Meth. Phys. Res. **A507**, 373 (2003).
[17] B.W.J. McNeil, G.R.M. Robb, M.W. Poole. Two-beam free-electron laser. Phys. Rev. **E 70**, 035501(R) (2004).
[18] S. Saviz, Z. Rezaei, F.M. Aghamir. Gain enhancement in two-stream free electron laser with a planar wiggler and an axial guide magnetic field. Chin. Phys. B **21**, 094103 (2012).
[19] V.V. Kulish et al. Superheterodyne plasma-beam lasers with helical electron beams. Applied Physics, No. 5, 24 (2014).
[20] O.V. Lysenko, J.J. Volk, A.S. Serozhko. Multiple three-wave resonances in two-stream free-electron laser with a helical electron beam. Inform. Telecomm. Sci. **9**, 58 (2018).
[21] E. Sabia, E.Di Palma, G. Dattoli. Two-Beam Free-Electron Lasers and Self-Injected Nonlinear Harmonic Generation. Appl. Sci. **11**, 6462 (2021).
[22] L. Gao et al. Study of multiple beam backward wave oscillator based on corrugated waveguide TWT. 2017 Eighteenth Int. Vacuum Electronics Conference (IVEC), pp. 1-2 (2017).
[23] Z. Zang et al. Multiple-beam and double-mode staggered double vane travelling wave tube with ultra-wide band. Sci. Rep. **10**, 20159 (2020).
[24] J. Luo et al. A 340 GHz High-Power Multi-Beam Overmoded Flat-Roofed Sine Waveguide Traveling Wave Tube. Electronics. **10**, 3018 (2021).
[25] W. Wang et al. Double-mode and double-beam staggered double-vane traveling-wave tube with high-power and broadband at terahertz band. Sci. Rep. **12**, 12012 (2022).
[26] K.N. Islam, E. Schamiloglu. Multiple electron beam generation with different energies and comparable currents from a single cathode potential for high power traveling wave tubes (TWTs). J. Appl. Phys. **131**, 044901 (2022).
[27] H. Seidfaraji et al. A multibeam metamaterial backward wave oscillator. Phys. Plasmas. **26**, 073105 (2019).
[28] Y. Zhang, Y. Zhou, L. Dong. THz radiation from two electron-beams interaction within a bi-grating and a sub-wavelength holes array composite sandwich structure. Optics Express. **21**, 21951 (2013).
[29] D.I. Trubetskov, A.V. Titov. Theory of Electron-Wave Devices for the Short-Wave Part of the Microwave Band. J. Comm. Tech. Electron. **59**, 784-791 (2014).
[30] The MAGIC user's manual. MRC/WDC-R-556 (2006).
[31] CST STUDIO SUITE 2017 Offers EM Simulation on Every Scale. Microwave J. February, 2 p. (2017).
[32] V.G. Baryshevsky, I.D. Feranchuk. Parametric beam instability of relativistic charged particles in a crystal. Phys. Let. A. **102**, 141 (1984).
[33] V.G. Baryshevsky. Spontaneous and induced radiation by electrons/positrons in natural and photonic crystals. Volume free electron lasers (VFELs): From microwave and optical to X-ray range. Nucl. Instr. Meth. Phys. Res. **B55**, 17 (2015).
[34] V.G. Baryshevsky, A.A. Gurinovich. Spontaneous and induced parametric and

Smith-Purcell radiation from electrons moving in a photonic crystal built from the metallic threads. Nucl. Instr. Meth. Phys. Res. **B252**, 92 (2006).

[35] V.G. Baryshevsky, K.G. Batrakov, I.Ya. Dubovskaya. Parametric (quasi-Cerenkov) X-ray free electron lasers. J. Physics D: Appl. Phys. **24**, 1250 (1991).

[36] K.G. Batrakov, S.N. Sytova. Modelling of Volume Free Electron Lasers. Comp. Math. Math. Phys. **45**, 666 (2005).

[37] W. Woo et al. Phase locking of highpower microwave oscillators. J. Appl. Phys. **65**, 861 (1989).

[38] H. Sze, D. Price, B. Harteneck. Phase locking of two strongly coupled vircators. J. Appl. Phys. **67**, 2278 (1990).

[39] N.M. Ryskin, V.N. Titov, O.V. Umantsiva. Phase Locking and Mode Switching in a Backward-Wave Oscillator With Reflections. IEEE Trans. Plasma Sci. **44**, 1270 (2016).

[40] A.B. Adilova, N.M. Ryskin. Influence of the delay on mutual synchronization of two couples gyrotrons. Radiophys. Quant. Electron. **63**, 703 (2021).

[41] S.N. Sytova. Methods of chaos control in radiation of charged particles moving in non-one-dimensional periodical structures. Nonlin. Phen. Compl. Syst. **20**, 144 (2017).

[42] S.N. Sytova. Comparison of one-dimensional and volume distributed feedback in microwave vacuum electronic devices. Nonlin. Phen. Compl. Syst. **15**, 378 (2012).