Nonlinear Phenomena in Complex Systems, vol. 20, no. 2 (2017), pp. 21 - 29

Methods of Chaos Control in Radiation of Charged Particles Moving in Non-One-Dimensional Periodical Structures

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(Received ?????03 December, 2016)

Methods of mathematical modeling were used for simulation of processes of radiation of charged particles, moving in different types of volume free electron lasers (VFEL). Generalized system for simulation of different types of VFEL is proposed. Some results of simulation of VFEL are given. Overview of methods of chaos control in VFEL is proposed. Such chaos control can be carried out via varying external electromagnetic waves in VFEL, changing of geometry of volume distributed feedback, parameters of electron beams etc.

PACS numbers: 41.60.Cr; **AMS Subject Classification:** 35Q60, 35F20, 65M06
Keywords: Volume Free Electron Laser, mathematical modelling, chaos

1. Introduction

The last fifty years we see rapid development of vacuum electronics in the world. During this time it was created a large number of different types of vacuum electronic devices, such as traveling wave tubes (TWT), backward wave tubes (BWT), free electron lasers (FEL), orotrons, masers, multiwave Cherenkov generators, etc. [1] – [4]. The basis of their work consists in the emission of electrons, grouped in bunches and interacting in resonator or undulator with slow electromagnetic waves. Such devices have a number of advantages [1], in particular, good efficiency as well as generation of powerful radiation in a narrow spectral range. Among the problematic aspects of their operation it should be called a risk of waveguide breakdown on high power and complexity when creating oversized systems with electron beam cross-section much larger than the wavelength. The last problem is connected with the necessity to use thin ribbon or cannular electron beams that interacts with electromagentic waves only at small distance of the resonator surface.

So, now we have a wide variety of military and commercial applications of vacuum electronic devices requiring high power at high frequency as well as reliable performance at such power with high efficiency and low cost. Let us emphasize that vacuum electronics amplifiers and oscillators are used in scientific research areas such as high-energy particle accelerators and plasma heating for controlled thermonuclear fusion. In medical systems they are used in compact radio-frequency accelerators and nuclear magnetic resonance spectrometers. Other applications are for commercial satellite communication systems, broadcasting, microwave ovens for industrial and home use etc.

In the same line with these devices we can put volume free electron lasers (VFEL) [5], [6] running on the radiation of relativistic electrons in two- and three-dimensional spatially-periodic structures in synchronism with one or more coupled strong electromagnetic waves. For these waves the conditions of Bragg diffraction near degeneration points of the roots of the dispersion equation should be valid. Dynamical diffraction provides volume (non-one-dimensional) distributed feedback (VDFB) in contrast to one-dimensional distributed feedback

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in TWT, BWT etc. VFEL spatially-periodic structures are natural crystals (in X-ray range [7]) or artificial electromagnetic (photonic) crystals with a period proportional to the radiation wavelength [6], [8]. Principles of VFEL functioning are valid for all frequency ranges and different mechanisms of spontaneous emission [5], [7]. VFEL lasing was obtained in millimeter range [9], [10] and put the beginning of its experimental development. In the microwave range VFEL experimental setup uses now grid and foil resonators [11], [12], having all necessary properties of photonic crystals.

Investigation of VFEL nonlinear stage can be carried out only by methods of mathematical modelling [13], [14]. The main distinguish between VFEL mathematical model and TWT, BWO models [3] is the following. In VFEL modelling [14] we use the method of averaging over initial phases of electron entrance in the resonator that takes into account as initial phase of an electron not only the moment of time t_0 as in other models [3] but also transverse spatial coordinate of an electron entrance in the resonator at $z = 0$. As a result, we managed to simulate strongly non-one-dimensional VFEL system with electron beam of broad cross-section as one-dimensional system.

Let us note that this one-dimensional system with one space coordinate z describes a two-dimensional or three-dimensional dynamical diffraction geometry, which, coupled with the simulation of a wide cross-section of the electron beam gives good results on the modeling of two-dimensional or three-dimensional VFEL systems [13] – [20].

In the literature, nonlinear dynamics and chaos in cited above vacuum electronic devices were studied in detail [3], [21] – [24]. VFEL nonlinear dynamics was investigated by methods of mathematical modelling too [13] – [20]. In VFEL, the reasons of initiation of chaotic dynamics remain the same as in other electronic devices: significant perturbation in movement of electrons and deformation of bunches leading to generation higher harmonics in the system and vice versa. A whole spectrum of external operating (bifurcational) parameters exists in VFEL. Their varying leads to qualitative changing of the system behavior. Such parameters are the following: electron beam current, length of the resonator, geometry parameters, factors of asymmetry etc. Investigation of chaos in VFEL is important in the light of experimental development of VFEL. In our previous works [13] – [20] a gallery of different chaotic regimes for VFEL laser intensity with corresponding phase space portrait, attractors and Poincaré maps was proposed.

The main goal of the paper presented is the further investigation of different aspects of chaotic nature of VFEL by methods of mathematical modelling. Here we propose the mostly common VFEL model taking into account dispersion of electromagnetic waves in the system and resonator with several sections with different parameters. Some different electron beams moving in resonator and external reflectors placed in the system are considered too.

Why multiple-beam VFEL? Multiple beams have been proposed for applications such as microwave tubes, FEL, heavy ion inertial fusion drivers and other cases where single beam systems may have difficulties [25] – [27]. Use of multiple beams permits higher total charge. Higher peak currents may be achievable with shorter wigglers and higher saturated powers. If the total charge is held fixed, then less charge is needed from each injector and lower emittance may be possible. So, multiple-beam VFEL can have analogous advantages [8], [6].

This article is arranged as follows. Section 2 describes VFEL generalized system of equations. Section 3 proposes mathematical model for two-beam two-wave VFEL. Some numerical results for simulation of this model are given in Section 4 with discussion of chaos control methods here.

2. Generalized system of VFEL equations

The most simple scheme of VFEL is depicted in Fig.1 [14]. Here an electron beam with electron velocity $\mathbf{u}$ passes through a spatially periodic resonator of the length L. This resonator can be natural or artificial photonic crystal in dependence of work wavelength. Under diffraction conditions two strong coupled electromagnetic waves are excited in the resonator. If simultaneously electrons of the beam are under synchronism condition, they emit electromagnetic radiation in directions depending on diffraction conditions.

Two-wave VFEL can work in two different geomentries. Bragg geometry is depicted in Fig.1, where transmitted and diffracted waves go out through opposite sides of resonator. In Laue geometry both waves go out through its back side at $z = L$. By assignment of system parameters, VFEL can operate in regimes of TWT and BWT, i.e. in one-dimensional geometry [20]. For three-wave diffraction we have three different geometries [15] and so on for multiwave cases.

In Fig.2 a multisection multiwave VFEL scheme with some electron beams in very general outline is proposed. Multiple beam free-electron lasers were proposed firstly [25]. Also it should be mentioned two-stream FEL [26] with slightly different velocities of electron beams. Multisection VFEL was considered in [11]. Additionally reflectors can be put along the edges of VFEL [14].

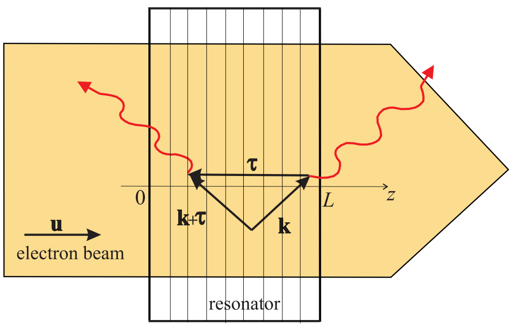

FIG. 1: Two-wave VFEL.

Let us depict all these VFEL variants by the generalized system of equations. First proposition of given below system was in [28].

The system of nonlinear equations for VFEL modelling is obtained from Maxwell equations in the slowly-varying envelope approximation. Electron beam dynamics is described by method of averaging over initial phases of electrons.

Let us consider VFEL resonator of the length L consisting from l sections with different parameters in the following domain Ω:

$$\Omega = \mathbf{G}(z) \bigcup \{-2\pi \le p \le 2\pi\} \bigcup \{t > 0\},$$

$$\mathbf{G}(z) = \bigcup_{i=1}^{l} G_i, \quad G_i = [z_i^1, z_i^2],$$

$$z_i^2 = z_{i+1}^1, \quad z_1^1 = 0, \quad z_l^2 = L.$$

Let us consider N electromagnetic waves in diffraction conditions with amplitudes $E_j(z,t)$ with n_i waves in each i-th section, $\sum_{i=1}^{l} n_i = N$. Here we have

$$j = \sum_{k=1}^{i-1} n_k + \mathrm{j}, \quad \mathrm{j} = 1, 2, ..., n_i.$$

Then let us consider $M = M^{left} + M^{right}$ electron beams with M^{left} beams coming from the left boundary $z = 0$ of the system and M^{right} ones coming from the right boundary at $z = L$. Each beam is described by the phase $\theta_m(t, z, p)$ of electrons with respect to electromagnetic wave and is calculated in the whole domain $\mathbf{G}$. In each section, it may be in synchronism with one wave with wave vector $\mathbf{k}_\alpha$. We do not consider the beam synchronizm with several waves since it is practically unachievable by the parameters of diffraction.

So, electromagnetic field for N-wave M-beam ($M \le N$) VFEL is represented in the common case in the following form:

$$\mathbf{E}(\mathbf{r}, t) = \sum_{j=1}^{N} \mathbf{e} E_j(z,t) e^{\imath(\mathbf{k}_j \mathbf{r} - \omega t)}, \qquad (1)$$

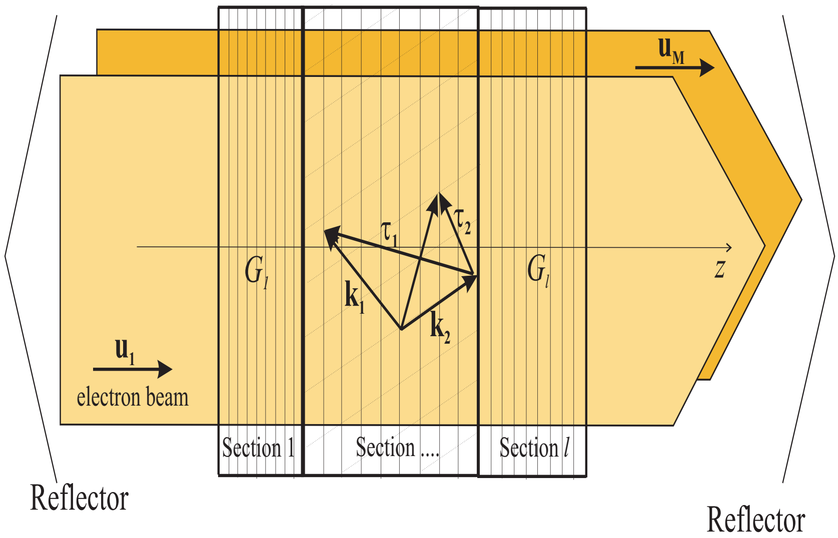

FIG. 2: Multiple beam multisection VFEL with external reflectors.

where $E_j(z,t)$ is the amplitude of the wave number j generated in the system with wave vector $\mathbf{k}_j$ and frequency ω.

Electron beams current is presented in the form:

$$\mathbf{j}(\mathbf{r},t) = \sum_{m=1}^{M} \mathbf{e} j_m(z,t) e^{\imath(\mathbf{k}_\alpha \mathbf{r} - \omega t)}, \qquad (2)$$

where $j_m(z,t)$ are amplitudes of expansion of m-th electron beam current. $\mathbf{e}$ is a vector of polarization. It can be the vector of σ-polarization $\mathbf{e}_\sigma$ or π-polarization $\mathbf{e}_\pi$. $\imath$ is the imaginary unit. Wave vector $\mathbf{k}_\alpha$ belong to the set of wave vectors $\mathbf{k}_j$ in the system.

Slowly-varying envelope approximation for all waves E from (1) means that:

$$|\frac{1}{k}\frac{\partial E}{\partial z}| \ll |E|, \quad |\frac{1}{\omega}\frac{\partial E}{\partial t}| \ll |E|, \qquad (3)$$

where $k = \omega/c$. So, second derivatives with respect to time and space can be neglected.

So, considering vector of amplitudes $\mathbf{E} = (E_1, E_2, ..., E_N)^T$ and vector of right-hand side of dimension N $\mathbf{I} = (I_1, I_2, ..., I_M, 0, ..., 0)^T$, the system of equations has the matrix form:

$$\mathbf{A}\frac{\partial \mathbf{E}}{\partial t} + \mathbf{B}\frac{\partial \mathbf{E}}{\partial z} + \mathbf{CE} = \mathbf{DI}. \qquad (4)$$

In (4) matrices are of the following type:

$$\mathbf{A} = \begin{pmatrix} 1+a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & 1+a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{N2} & \cdots & 1+a_{NN} \end{pmatrix},$$

$$\mathbf{B} = \begin{pmatrix} b_{11} & 0 & \cdots & 0 \\ 0 & b_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_{NN} \end{pmatrix},$$

$$\mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1N} \\ c_{21} & c_{22} & \cdots & c_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N1} & c_{N2} & \cdots & c_{NN} \end{pmatrix}.$$

Vector $\mathbf{I}$ has the following components:

$$I_m = \Upsilon_m \int_0^{2\pi} (2\pi - p) \times \left(e^{-\imath\Theta_m(t,z,p)} + e^{-\imath\Theta_m(t,z,-p)} \right) dp, \qquad (5)$$

$m = 1, 2, \ldots, M$.

The matrix $\mathbf{A}$ differs from identity matrix. $\mathbf{A}$ contains additional terms a_{ij} which determine dispersion of electromagnetic wave in the resonator. Diagonal matrix $\mathbf{B}$ includes direction cosines of wave vectors $\mathbf{k}_i$. Matrix $\mathbf{C}$ describes dynamical diffraction in the system. Matrix $\mathbf{D}$ contains unities and zeros (beam interacts or do not interact with present electromagnetic wave).

In our model $m-$th electron beam is simulated via phases of m-th electron beam relative to electromagnetic wave $\Theta_m(t,z,p)$ ($m = 1,2,...,M$) in the following way [14]:

$$\frac{\partial^2\Theta_m(t,z,p)}{\partial z^2} = \Psi_m\left(k_{\alpha z} - \frac{\partial\Theta_m(t,z,p)}{\partial z}\right)^3$$

$$\times\mathrm{Re}\left(E_\alpha(t-z/u_m,z)e^{i\Theta_m(t,z,p)}\right). \qquad (6)$$

All above $t > 0$, $z \in [0, L]$, $p \in [-2\pi, 2\pi]$.

Boundary conditions for amplitudes of electromagnetic waves can be written as follows:

$$\mathbf{E}(t,\mathbf{\Gamma}_1) = \mathbf{E^0}(t) + \mathbf{F}\mathbf{E}(t,\mathbf{\Gamma}_2). \qquad (7)$$

The vector $\mathbf{E^0}$ with components E_i^0 (some of them can be equal to zero) contains amplitudes of the external electromagnetic waves incident on boundaries $z = 0$ or $z = L$. The matrix $\mathbf{F}$ determines connections between separate sections of VFEL and reflectors. The form of its components can be found in [14].

Components of vectors $\mathbf{\Gamma_1}$ and $\mathbf{\Gamma_2}$ for wave with amplitude E_j are coordinates of entrance and leaving of this wave i-th section: z_i^1 or z_i^2 or vice versa in dependence of wave spread direction in the current section.

Boundary conditions for phases of all electron beams are the next:

$$\Theta_l(t,\Gamma_1^m,p) = p, \quad \frac{\partial\Theta(t,\Gamma_1^m,p)}{\partial z} = k_{mz}^0 - \omega/u_m, \qquad (8)$$

$m = 1,\ldots,M$.

Component ${\Gamma_1}^m$ is equal to 0 or L in dependence of electron beam entrance side (from the left or from the right). k_{mz}^0 corresponds to projection on axis z of electromagnetic wave vector in synchronism with electron beam in the first section of beam entrance.

3. Two-wave two-beam VFEL

For two-wave two-beam VFEL when one beam is in resonance with the transmitted wave E and the second one – with diffracted wave E_τ considering regime of generator without dispersion components, we can obtain the following form of matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$.

$$\mathbf{A} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix}b_{11} & 0\\ 0 & b_{22}\end{pmatrix},$$
$$\mathbf{C} = \begin{pmatrix}c_{11} & c_{12}\\ c_{21} & c_{22}\end{pmatrix}, \qquad \mathbf{I} = \begin{pmatrix}I_1\\ I_2\end{pmatrix},$$
$$\mathbf{D} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.$$

So, this system in full is the next:

$$\frac{\partial E}{\partial t} + \gamma_0 c\frac{\partial E}{\partial z} + 0.5i\omega l_1 E - 0.5i\omega\chi_\tau E_\tau = I_1,$$

$$\frac{\partial E_\tau}{\partial t} + \gamma_\tau c\frac{\partial E_\tau}{\partial z} - 0.5i\omega\chi_{-\tau}E + 0.5i\omega l_2 E_\tau = I_2,$$

$$\frac{\partial^2\Theta_1(t,z,p)}{\partial z^2} = \frac{e\Phi_1}{m\gamma_1^3\omega^2}\left(k_z - \frac{\partial\Theta_1(t,z,p)}{\partial z}\right)^3.$$

$$\mathrm{Re}\left(E(t-z/u_1,z)e^{i\Theta_1(t,z,p)}\right),$$

$$\frac{\partial^2\Theta_2(t,z,p)}{\partial z^2} = \frac{e\Phi_2}{m\gamma_2^3\omega^2}\left(k_{\tau z} - \frac{\partial\Theta_2(t,z,p)}{\partial z}\right)^3.$$

$$\mathrm{Re}\left(E_\tau(t-z/u_2,z)e^{i\Theta_2(t,z,p)}\right),$$

$$\frac{\partial\Theta_1(t,0,p)}{dz} = k_z - \omega/u_1, \quad \Theta_1(t,0,p) = p,$$

$$\frac{\partial\Theta_2(t,L,p)}{dz} = k_{\tau z} - \omega/u_2, \quad \Theta_2(t,L,p) = p,$$

$$I_{1,2} = \Upsilon_{1,2}\int_0^{2\pi}(2\pi - p)(e^{-i\Theta_{1,2}(t,z,p)}$$

$$+e^{-i\Theta_{1,2}(t,z,-p)})dp,$$

$$E(0,t)=0, \quad E_\tau(L,0)=0.$$

Here E and E_τ are amplitudes of transmitted and diffracted waves, respectively. $\gamma_{0,\tau}$ are diffraction cosines. System parameters are the following: $l_{0,\tau} = (k^2_{\cdot,\tau}c^2 - \omega^2\varepsilon_0)/\omega^2$, $l_1 = l_0 + \delta_1$, $l_2 = l_\tau + \delta_2$. $\delta_{1,2}$ are detuning from the Cherenkov condition for both beams. $\gamma_{1,2}$ are Lorenz-factors for corresponding beam velocities $u_{1,2}$ and densities $n^b_{1,2}$. ε_0 is a mean dielectric susceptibility and $\chi_{\pm\tau}$ are Fourier components of the dielectric susceptibility of the target. $\Upsilon_{1,2} = en^b_{1,2}u_{1,2}\Phi_{1,2}/(4\pi)$, $\Phi_{1,2} = \sqrt{l_{0,\tau} + \chi_0 - c^2/(u_{1,2}\gamma_{1,2})^2}$.

4. Results of numerical experiments

Numerical methods for VFEL simulation were developed and used effectively for their different types [13]–[20]. These methods allow to use parallel processing effectively.

Let us consider here results of simulation of two-wave two-beam VFEL with parameters close to VFEL exprerimental setup [11].

In Fig. 3 we vary current density of the first electron beam j_1 from 100 to 250 A/cm^2 at fixed j_2=2 kA/cm^2. Let us note, that in the absence of the second beam the threshold current density in the system is equal j_1=300 A/cm^2. In the absence of the first one the threshold current density of the second beam is equal to j_2=6 kA/cm^2. So, using two beams allows considerably decrease the current thresholds. Moreover, one can see considerable changing of the type of solution with changing of currant values. Analogous examples were obtained changing second beam at the fixed first one. So, this can be considered as one way of chaos control in VFEL.

Below we consider one-beam VFEL. So, other way of chaos control in VFEL can be realized via varying external electromagnetic waves in VFEL. In this paper we consider the

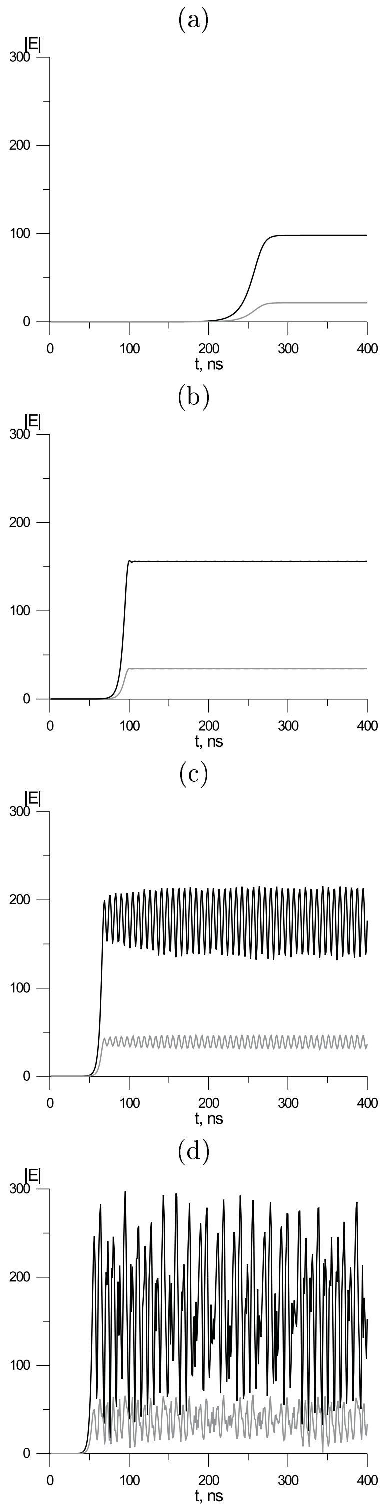

FIG. 3. Amplitudes of transmitted (grey lines) and diffracted (black lines) waves for $j_1 = 100$ A/cm^2 (a), j_1=150 A/cm^2 (b), j_1=200 A/cm^2 (c),j_1=250 A/cm^2 (d).

influence of external incident wave $E_0(t)$ at $z = 0$. The parameters of the system correspond to parameters of VFEL with a grid resonator [11]. Various types of waves $E^0(t)$ were considered. Here we give only two cases: $E^0(t) = 100$ (black curves in Fig.4) and $E^0(t) = 100+20sin(3t)+20a$ (grey curves), where a is a random number in the interval $[0, 1]$. The results of simulation for transmitted waves at $z = L$ are shown in Fig.4b-d for different parameter of diffraction geometry τ_x.

It is obvious, that the chaotic component introduced by the generator of random numbers is suppressed when we deal with periodic regimes (Fig.4b and c)). Moreover in chaotic regime (Fig.4d)) without such suppression should have been substantially more "shaggy". This is the manifestation of one of the main VFEL properties – the suppression of parasitic modes in the system.

Besides traditional changing of system parameters one another way of chaos control in VFEL also can be realized via changing VDFB geometry. In [20] it was shown that variation from one-dimensional to non-one-dimensional geometry leads to changing in the type of dynamical solution. So, the choice of VFEL geometry can implement periodic dynamics rather chaotic regime.

5. Conclusions

We propose a generalizes system of equations describing the various options for multiple beam multiwave VFEL. It takes into account multi-section resonators, the dispersion of electromagnetic waves in the system, external reflectors, etc. Mathematical modeling of two-beam two-wave VFEL was carried out using proposed system of equations. It was shown that changing the electron beam current density leads to change VFEL chaotic dynamics and can significantly reduce the threshold current values in the system. It is also one of the mechanisms of chaos control in the system.

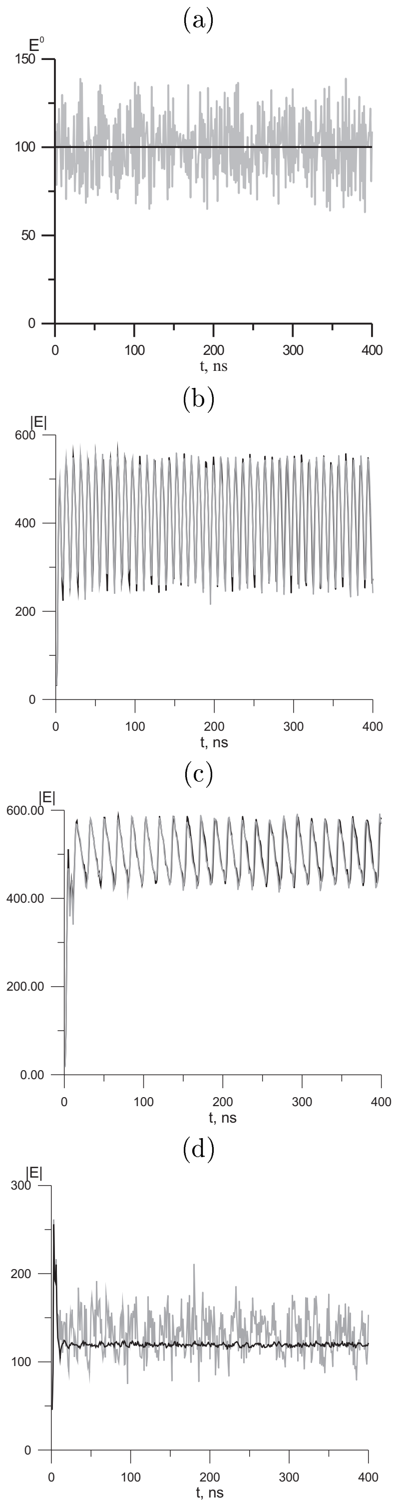

FIG. 4. Suppression of chaotic component introduced by the random number generator. a: External incident waves at $z = 0$ $E^0(t) = 100$ (black curve) and $E^0(t) = 100 + 20sin(3t) + 20a$ (grey curve). b, c, d: Output for corresponding transmitted waves at $z = L$ for $\tau_x = 1.0$, $\tau_x = 1.1$, $\tau_x = 1.4$, respectively.

The way of chaos control in VFEL can be occurred via changing system parameters, geometry of volume distributed feedback, via external electromagnetic waves in VFEL, parameters of electron beams etc. Changing these VFEL parameters leads to changes in the type of dynamic chaotic solutions and by the proper choice of parameters one can realized VFEL periodic dynamics rather chaotic one and vice versa.

The influence of external electromagnetic waves on the nature of the VFEL generation demonstrates that by a special choice of the parameters of external signals one can obtain higher values of the field amplitudes as well as a fundamentally different type of solution.

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