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Angular distribution of photons from channelled particles

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Abstract. It is shown that photons emitted by channelled particles form a typical diffraction pattern which contains information about the crystal structure. It is also shown that the orientational dependence of the angular distribution of the photons produced by the channelled particles under diffraction conditions has a sharp peak whose angular value is much less than the Lindhard angle. It is determined by the structure amplitudes such as those in the case of the dynamic diffraction of γ quanta in a crystal.

1. Introduction

Recently, considerable success in the field of the crystal structure investigation has been achieved by using new types of radiation sources, for example, synchrotron radiation (Rouv and Wivsr 1978). Nowadays, x-rays and γ radiation accompanying the passage of electrons (or positrons) through crystals are widely studied both theoretically and experimentally (Kalashnikov 1981, Ter-Mikaelian 1972). This radiation can also be used successfully for crystallographic and spectroscopic investigations. Moreover, experiments with such radiation have some advantages. The contributions to this radiation are made by various radiation mechanisms: e.g. coherent bremsstrahlung, the parametric radiation resulting from the coherent excitation of crystal atoms by particles passing through the crystal (Baryshevsky and Feranchuk 1976) and the radiation caused by radiative transitions between the transverse energy zones of channelled particles (Baryshevsky and Dubovskaya 1977, Kumakhov 1977, Kalashnikov 1981). The latter mechanism has been investigated experimentally by several groups (Vorobiev *et al* 1975, Pantel and Alguard 1979, Andersen *et al* 1981).

According to Baryshevsky and Dubovskaya (1976, 1977), a channelled particle making the transition between the zones of the transverse energy can be considered as an analogue of a relativistic atom. The frequency of the photon emitted by such an atom depends essentially on the particle energy and on the refractive properties of the crystal caused by the Doppler effect. The analogy between the channelled particle and the relativistic atom suggests that many phenomena well known in atomic physics can be observed in the radiation process considered above. In particular, the angular distribution of the emitted photons depends essentially on the refractive properties of the medium.

It is well known that the radiation from relativistic particles is concentrated within a narrow angle relative to the direction of particle motion. However, it has been shown by Baryshevsky *et al* (1978, 1980) that the diffraction of the radiated photons results in γ quanta propagating at a large angle with respect to the particle's direction of motion and the formation of a typical diffraction pattern which depends on the crystal structure. The intensity of this radiation is found to be high: an electron with energy 20–40 MeV radiates $N_\gamma \sim 10^7$ γ quanta within a diffraction peak with energies in the range 10–100 keV provided that the particle current is 10^{-6} A, the crystal thickness is $L \sim 10^{-2}$ cm and the radiation angle is $\Delta\vartheta \sim 10^{-6}$ rad. This means that the intensity of such a source essentially exceeds the intensity of an ordinary x-ray source in the same angular and spectral intervals.

It should be noted that a diffraction pattern of the radiation from channelled particles will be formed practically under arbitrary conditions both for planar and for axial channelling if the frequencies of photons radiated by the channelled particles lie in the spectral range $\omega \sim$ 10–100 keV. This statement follows immediately if we imagine the channelling radiation as coming from a certain source whose angular divergence of radiation is $\Delta\vartheta$ (a 'spotlight') moving inside the crystal. It is well known that such an x-ray source produces a diffraction pattern.

Let us consider, for example, one of the simplest cases (see figure 1). According to Swent *et al* (1979), an electron with energy $E = 28$ MeV radiates γ quanta in the energy range 10–200 keV in Si. Because a photon beam is concentrated at an angle of $\vartheta \sim m/E$, i.e. $\vartheta \ll \theta_d$ (θ_d is the angle between $\boldsymbol{k} + 2\pi\boldsymbol{\tau}$ and $\boldsymbol{k}$, where $\boldsymbol{k}$ is the photon wavevector and $2\pi\boldsymbol{\tau}$ is the reciprocal lattice vector which characterises the family of crystallographic planes on which the photon diffraction occurs) we can believe that the wavevector $\boldsymbol{k}$ is approximately parallel to the z axis. In this case (see figure 1) photons with frequencies satisfying the relation $|\boldsymbol{k}| = \frac{1}{2}[(2\pi\tau_x)^2 + (2\pi\tau_z)^2](2\pi\tau_z)^{-1}$ will be diffracted by the family of crystallographic planes corresponding to the vector $2\pi\boldsymbol{\tau} = ((2\pi/a)n_1, 0, (2\pi/a)n_3)$. It is evident that Laue diffraction will take place if $|\boldsymbol{k}| = (\pi/a)[n_3 + (n_1^2/n_3)]$. The diffraction peak will be observed at the angle $\theta_d = 2n_1n_3(n_3^2 + n_1^2)^{-1}$ which is much greater than the radiation angle ϑ and the Lindhard angle of channelling.

According to figure 1, it is possible to observe only the asymmetric Laue case of

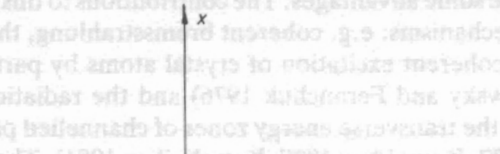

Figure 1. Kinematics of the radiation process in diffraction.

diffraction for a channelled particle incident almost normal to the crystal surface. However, symmetric Laue diffraction can be obtained for a channelled particle incident at an oblique angle relative to the crystal surface. In this case the crystallographic planes (axes) between which the particle oscillates are directed at the angle $\psi \neq \pi/2$ relative to the crystal surface. This situation is different from that in ordinary channelling experiments where crystallographic planes (axes) are always chosen normal to the target surface.

The present paper shows that the radiation emitted by the channelled particle in the x-ray region always produces a certain diffraction pattern. This means that the radiation at a large angle relative to the particle's direction of motion can be observed in addition to the radiation in a forward direction.

The orientational dependence of the angular distribution of photons produced by channelled particles undergoing diffraction has a sharp peak whose angular value is much less than the Lindhard angle. It is determined by the structure amplitudes such as those in the case of the dynamic diffraction of γ quanta in the crystal. This gives us the opportunity to obtain information about the structure of the crystal directly from analysis of the spectral angular distribution.

2. Wavefunctions of the channelled particle and the photon

The formula determining the transition probability per unit time for the process in which a particle scattered in a constant field produces some new particles is well known (Landau and Lifshitz 1968, p 276). If photons are produced in this situation the cross-section can be written in the form

$$d\sigma = 2\pi\delta(E - E_1 - \omega)|M(\boldsymbol{p}_1, \boldsymbol{k}; \boldsymbol{p})|^2 \frac{d^3 p_1\, d^3 k}{8(2\pi)^6 p E_1 \omega} \tag{1}$$

where

$$M(\boldsymbol{p}_1, \boldsymbol{k}; \boldsymbol{p}) \equiv e(4\pi)^{1/2} M_{if} = e(4\pi)^{1/2} \int d\boldsymbol{r}\, \psi_{p_1}^{(-)*}(\boldsymbol{r})\, (\boldsymbol{\alpha} \boldsymbol{A}_{ks}^{(-)*}(\boldsymbol{r}))\, \psi_p^{(+)}(\boldsymbol{r}). \tag{2}$$

$\psi^{(\pm)}(\boldsymbol{r})$ are the explicit solutions of the Dirac equation describing the scattering process of the electron (positron) in the crystal, s is the index of the photon polarisation, $\boldsymbol{k}$ (ω) is the wavevector (frequency) of the photon. $\boldsymbol{k} = (\boldsymbol{k}_\perp, k_z \boldsymbol{n})$ where $\boldsymbol{k}_\perp$ and k_z are the transverse and longitudinal components of $\boldsymbol{k}$. $\boldsymbol{p}$ and $\boldsymbol{p}_1$ are the particle momenta in the initial and final states, E and E_1 are their energies, $\boldsymbol{\alpha}$ is the Dirac matrix and $A_{ks}^{(-)}(\boldsymbol{r})$ is the photon wavefunction with the asymptotic form of a plane wave plus the incoming spherical wave.

It has been shown by Baryshevsky and Dubovskaya (1977) that, if we consider the radiation from the particles in the crystal, the functions $\psi_{p(p_1)}^{(\pm)}(\boldsymbol{r})$ can be written in the same form as for the process of bremsstrahlung in a screening Coulomb potential, i.e. (Olsen and Maximon 1959)

$$\begin{aligned} \psi_p^{(+)}(\boldsymbol{r}) &= \exp(i\boldsymbol{p}\boldsymbol{r})[1 - (i/2E)(\boldsymbol{\alpha}\nabla)]\, \varphi_p^{(+)}(\boldsymbol{r})\, U_p \\ \psi_{p_1}^{(-)}(z) &= \exp(i\boldsymbol{p}_1\boldsymbol{r})[1 - (i/2E_1)(\boldsymbol{\alpha}\nabla)]\, \varphi_{p_1}^{(1)}(\boldsymbol{r})\, U_{p_1} \end{aligned} \tag{3}$$

where $U_{p(p_1)}$ are four-component spinors satisfying the Dirac equation for a free particle with momentum $\boldsymbol{p}$ ($\boldsymbol{p}_1$). The functions $\exp(i\boldsymbol{p}_{(1)}\boldsymbol{r})\, \varphi_{p(p_1)}^{(\pm)}(\boldsymbol{r})$ satisfy equations of the

Schrödinger type:

$$
\begin{aligned}
&(\Delta + p^2 - 2EV(\boldsymbol{r}))\exp(\mathrm{i}\boldsymbol{p}\boldsymbol{r})\,\varphi_p^{(+)}(\boldsymbol{r}) = 0\\
&(\Delta + p_1^2 - 2E_1V(\boldsymbol{r}))\exp(\mathrm{i}\boldsymbol{p}_1\boldsymbol{r})\,\varphi_{p_1}^{(-)}(\boldsymbol{r}) = 0
\end{aligned}
\tag{4}
$$

where $V(\boldsymbol{r})$ is the potential determining the particle interaction with the crystal.

Taking into account the condition of matching the wavefunction at the crystal surface, the electron wavefunction for the initial and final states can be represented inside the crystal in the form

$$
\begin{aligned}
&\varphi_p^{(+)}(\boldsymbol{r}) = N_\perp^{-1/2}\sum_i C_{i\kappa}(\boldsymbol{p}_\perp)\,\varphi_{i\kappa}(\boldsymbol{\rho})\exp(\mathrm{i}p_{z_i}z)\\
&\varphi_{p_1}^{(-)}(\boldsymbol{r}) = N_\perp^{-1/2}\sum_f C^*_{f\kappa_1}(\boldsymbol{p}_{1\perp})\,\varphi_{f\kappa_1}(\boldsymbol{\rho})\exp(\mathrm{i}p_{1z_f}z)\\
&p_{z_i} = (p^2 - p_\perp^2 - p_{\perp i}^2)^{1/2} \qquad p_{1z_f} = (p_1^2 - p_{1\perp}^2 - p_{1\perp f}^2)^{1/2}\\
&C_{i\kappa}(\boldsymbol{p}_\perp) = (N_\perp/\Delta)^{1/2}\int_\Delta \exp(\mathrm{i}\boldsymbol{p}_\perp\boldsymbol{\rho})\,\varphi^*_{i\kappa}(\boldsymbol{\rho})\,\mathrm{d}\boldsymbol{\rho}\\
&C_{f\kappa_1}(\boldsymbol{p}_{1\perp}) = (N_\perp/\Delta)^{1/2}\int_\Delta \exp(\mathrm{i}\boldsymbol{p}_{1\perp}\boldsymbol{\rho})\,\varphi^*_{f\kappa_1}(\boldsymbol{\rho})\,\mathrm{d}\rho.
\end{aligned}
\tag{5}
$$

p_z and p_{1z} are the longitudinal components of the electron momentum, $\boldsymbol{p}_\perp = (p_x, p_y)$ and $\boldsymbol{p}_{1\perp} = (p_{1x}, p_{1y})$ are the transverse components of the electron momentum, and $\varphi_{i\kappa}(\boldsymbol{\rho})$ and $\varphi_{f\kappa_1}(\boldsymbol{\rho})$ are the two-dimensional Bloch functions satisfying the equations

$$
\left(-\frac{\hbar^2}{2E}\Delta_\rho + U(\boldsymbol{\rho})\right)\varphi_{i\kappa}(\boldsymbol{\rho}) = \mathscr{E}_{i\kappa}\varphi_{i\kappa}(\boldsymbol{\rho})
$$

and

$$
\left(-\frac{\hbar^2}{2E_1}\Delta_\rho + U(\boldsymbol{\rho})\right)\varphi_{f\kappa_1}(\boldsymbol{\rho}) = \mathscr{E}_{f\kappa_1}\varphi_{f\kappa_1}(\boldsymbol{\rho}).
$$

Correspondingly, $\boldsymbol{\kappa}$ and $\boldsymbol{\kappa}_1$ are the quasi-momenta of the particle within the first Brillouin zone, i and f are the numbers of particle transverse energy zones, $p_{\perp i}^2 = 2m\mathscr{E}_{i\kappa}$, $p_{1\perp f}^2 = 2m\mathscr{E}_{f\kappa_1}$, $\mathscr{E}_{i\kappa}$ and $\mathscr{E}_{f\kappa_1}$ are the energy eigenvalues of the above equations, Δ is the square of the crystal elementary cell, the origin of the frame of axes is situated on the left surface of the crystal target, and L is the target thickness along the z axis.

It should be noted that the wavefunction $\varphi_p^{(-)}(\boldsymbol{r})$ can be obtained from $\varphi_p^{(+)}(\boldsymbol{r})$ by means of the relation $\varphi_p^{(-)}(\boldsymbol{r}) \equiv [\varphi_{-p}^{(+)}(\boldsymbol{r})]^*$. However, while using the condition of matching for the wavefunction $\varphi_p^{(-)}(\boldsymbol{r})$ we can assume that the amplitude of an incident wavefunction is equal to unity at the crystal surface, $z = L$.

The photon wavefunction describing the Laue diffraction ($k_z > 0$, $k_{\tau_z} = k_z + 2\pi\tau_z > 0$) can be written in the two-beam approximation as

$$
\begin{aligned}
\boldsymbol{A}_{ks}^{(-)}(\boldsymbol{r}) = -\boldsymbol{e}_s^*\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})\{&\xi_{1s}^{0*}\exp[-\mathrm{i}(\omega/\gamma_0)\varepsilon_{1s}^*(L-z)]\\
&+\xi_{2s}^{0*}\exp[-\mathrm{i}(\omega/\gamma_0)\varepsilon_{2s}^*(L-z)]\}\\
&+\boldsymbol{e}_s^{\tau*}\beta_1\exp(\mathrm{i}\boldsymbol{k}_\tau\boldsymbol{\tau})\{\xi_{1s}^{\tau*}\exp[-\mathrm{i}(\omega/\gamma_1)\varepsilon_{1s}^*(L-z)]\\
&+\xi_{2s}^{\tau*}\exp[-\mathrm{i}(\omega/\gamma_1)\varepsilon_{2s}^*(L-z)]\}.
\end{aligned}
\tag{6}
$$

The photon function as well as the particle one is written only for the region inside the crystal, $\boldsymbol{k}_\tau = \boldsymbol{k} + 2\pi\boldsymbol{\tau}$, $\gamma_0 = \cos\theta = (1/\omega)(\boldsymbol{k}\boldsymbol{n}_z)$, ω is the photon frequency, $\boldsymbol{n}_z$ is the unit vector in the z direction, $\gamma_1 = (1/\omega)(\boldsymbol{k}_\tau \boldsymbol{n}_z)$, $\beta_1 = k_z/k_{\tau z}$, and $\boldsymbol{e}_s$ and $\boldsymbol{e}_s^\tau$ are the vectors defining the photon polarisation of the primary and diffracted waves, respectively. These vectors satisfy the conditions

$$(\boldsymbol{e}_s\boldsymbol{k}) = (\boldsymbol{e}_s^\tau \boldsymbol{k}_\tau) = 0 \qquad s = \pi, \sigma \qquad \boldsymbol{e}_\sigma \| \boldsymbol{e}_\sigma^\tau \| [\boldsymbol{k}2\pi\boldsymbol{\tau}]$$

$$\boldsymbol{e}_\pi \| [\boldsymbol{k}[\boldsymbol{k}2\pi\boldsymbol{\tau}]] \qquad \boldsymbol{e}_\pi^\tau \| [\boldsymbol{k}_\tau[\boldsymbol{k}2\pi\boldsymbol{\pi}]]$$

$$\xi_{1,2s}^0 = \mp\frac{2\varepsilon_{2,1s} - g_{0,0}}{2(\varepsilon_{2s} - \varepsilon_{1s})} \qquad \xi_{1,2s}^\tau = \mp\frac{g_{0,1}^s}{2(\varepsilon_{2s} - \varepsilon_{1s})}$$

where the negative sign relates to the first index and the positive sign to the second index,

$$\varepsilon_{1,2s} = \tfrac{1}{4}\{g_{00} + \beta_1 g_{11} - \beta_{1\alpha} \pm [(g_{00} + \beta_1 g_{11} - \beta_{1\alpha})^2 + 4\beta_1(\alpha g_{00} - g_{00}g_{11} + g_{10}^s g_{01}^s)]^{1/2}\}$$

$$\alpha = \frac{2(\boldsymbol{k}2\pi\boldsymbol{\tau}) + (2\pi\boldsymbol{\tau})^2}{\omega^2}.$$

The coefficients $g_{\alpha\beta}^s$ are defined by the series expansions of the dielectric constant of the crystal versus the reciprocal lattice vector (Afanasiev and Kagan 1965, Batterman and Cole 1964).

3. Differential cross section of the radiation

By substituting the wavefunctions (5) and (6) into expression (1) and making the corresponding transformations we obtain the following expression for the number of photons propagating at a given angle and with frequencies in a given spectral interval (in this case the relation $\omega \ll E$ is always satisfied; as a result the dipole approximation is used):

(*a*) For photons propagating in a forward direction

$$\mathrm{d}N_{\omega s} = \frac{e^2\omega\,\mathrm{d}\omega\,\mathrm{d}\Omega}{4\pi^2}\sum_{if} Q_{ii}|\boldsymbol{e}_s\boldsymbol{G}_{if}|^2 \left|\sum_{\mu=1,2}\xi_{\mu s}^0\frac{1-\exp(-\mathrm{i}q_{z_{if}}^{\mu s}L)}{q_{z_{if}}^{\mu s}}\right|^2 \tag{7}$$

where

$$q_{z_{if}}^{\mu s} = p_{z_i} - p_{1z_f} - k_z - \frac{\omega\varepsilon_{\mu s}}{\gamma_0} \simeq \omega(1 - v_\| \cos\theta) - \Omega_{if} - \frac{\omega}{\gamma_0}\mathscr{E}_{\mu s}.$$

θ is the angle between the photon wavevector $\boldsymbol{k}$ and the z axis, $\Omega_{if} = \mathscr{E}_{i\kappa} - \mathscr{E}_{f\kappa}$, Ω_{if} is the transition frequency in the laboratory frame, and

$$\boldsymbol{G}_{if} = -\mathrm{i}[v_\| \boldsymbol{n}_z(\boldsymbol{k}_\perp \boldsymbol{\rho}_{if}) + \Omega_{if}\boldsymbol{\rho}_{if}].$$

It should be noted that the function $\boldsymbol{G}_{if}$ is connected with the function $\boldsymbol{W}_{if}$ introduced by us (Baryshevsky and Dubovskaya 1977) as $\boldsymbol{G}_{if} = (1/E)\boldsymbol{W}_{if}$ and the general expression for $\boldsymbol{W}_{if}$ is given by Baryshevsky *et al* (1979, 1980). Also

$$\boldsymbol{\rho}_{if} = \int_\Delta \varphi_{i\kappa}(\boldsymbol{\rho})\boldsymbol{\rho}\varphi_{f\kappa}^*(\boldsymbol{\rho})\,\mathrm{d}\boldsymbol{\rho}.$$

It is sometimes convenient to represent the function $\boldsymbol{G}_{if}$ through the matrix element of the particle momentum. In this case

$$G_{if} = -(1/E)[(v_\parallel/\Omega_{if})\boldsymbol{n}_z(\boldsymbol{k}_\perp \boldsymbol{p}_{if}) + \boldsymbol{p}_{if}]$$

and

$$\boldsymbol{p}_{if} = -\mathrm{i}\int_\Delta \varphi_{i\kappa}(\boldsymbol{\rho})\boldsymbol{\nabla}_\rho \varphi^*_{f\kappa}(\boldsymbol{\rho})\,\mathrm{d}\boldsymbol{\rho}.$$

(*b*) For photons propagating at a large angle relative to the particle's direction of motion:

$$\mathrm{d}N^\tau_{\omega s} = \frac{e^2\beta_1\omega\,\mathrm{d}\omega\,\mathrm{d}\Omega}{4\pi^2}\sum_{if} Q_{ii}|e^\tau_s G_{if}|^2 \left|\sum_{\mu=1,2}\xi^\tau_{\mu s}\frac{1-\exp(-\mathrm{i}q^{\mu s}_{zif}L)}{q^{\mu s}_{zif}}\right|^2 \tag{8}$$

where

$$q^{\mu s}_{zif} = p_{z_i} - p_{1z_f} - k_{z\tau} - \frac{\omega\varepsilon_{\mu s}}{\gamma_1} \simeq \omega(1 - v_\parallel\cos\vartheta) - \Omega_{if} - \frac{\omega\varepsilon_{\mu s}}{\gamma_1}.$$

$Q_{ii} = |C_{i\kappa}(\boldsymbol{p}_\perp)|^2$ is the probability of the transverse energy state i being populated, ϑ is the angle between the vector $\boldsymbol{k}_\tau$ and the z axis (we use the system $\hbar = c = 1$).

For the frequencies or the angles for which the Bragg condition is not satisfied, the expression for the radiation cross section is essentially simplified (see Baryshevsky and Dubovskaya 1977, Baryshevsky *et al* 1978). It can easily be shown that $\mathrm{d}N^\tau_\omega \to 0$ because $\xi^\tau_{\mu s}\to 0$ and $\xi^0_{\mu s}\to 1$. As a consequence, equation (7) is rewritten as

$$\begin{aligned}\mathrm{d}N_\omega = \frac{e^2\omega\,\mathrm{d}\omega\,\mathrm{d}\Omega}{4\pi^2}\sum_{if} Q_{ii}(\Omega^2_{if}|\rho_{x_{if}}\sin\psi - \rho_{y_{if}}\cos\psi|^2 \\ + (v_\parallel\omega\sin^2\theta - \Omega_{if}\cos\theta)^2|\rho_{x_{if}}\cos\psi \\ + \rho_{y_{if}}\sin\psi|^2)\left|\frac{1-\exp(-\mathrm{i}q_{x_{if}}L)}{q_{z_{if}}}\right|^2\end{aligned} \tag{9}$$

where ψ is the azimuthal angle of the wavevector $\boldsymbol{k}$, $\rho_{x_{if}}$ and $\rho_{y_{if}}$ are the x and y components of the coordinate matrix element and are complex in the general case.

For a sufficiently thick crystal it is possible to replace $|[1-\exp(-\mathrm{i}q_{z_{if}}L)]/q_{z_{if}}|^2$ in equation (9) by $2\pi L\delta(q_{z_{if}})$, $q_{z_{if}} \simeq \omega(1 - v_\parallel\cos v) - \Omega_{if}$.

4. Angular distribution of the radiation

The cross section of the radiation can also be simplified when the energy of the radiating particle satisfies the condition $1 - v_\parallel \gg (1/\gamma_1)\,\mathrm{Re}\,\varepsilon_{\mu s}$. On the other hand, it is obvious that the particle energy must satisfy the diffraction condition for the emitted photons, i.e. $\omega_\mathrm{m} \simeq \Omega_{if}(1 - v_\parallel)^{-1} > \omega_\mathrm{d}$ where ω_d is the frequency for which the diffraction condition is satisfied. In this case we can suppose with high precision that the observed frequency of the photon does not depend on the dielectric properties of the crystal target. It is determined only by the radiation angle θ and by the frequency of the corresponding transition Ω_{if}, i.e.

$$\omega^{\mu s}_{if} \simeq \omega_{if} = \frac{\Omega_{if}}{1 - v_\parallel\cos\theta}. \tag{10}$$

In the case of planar channelling the angular distribution of the radiation in the dipole approximation can easily be calculated by performing the integration over frequencies using a δ function. As a result, we obtain the following expressions for the case of Laue diffraction.

(*a*) For radiation propagating at a large angle relative to the particles direction of motion:

$$\frac{dN_s^\tau}{d\Omega} = \frac{e^2 L \beta_1^2}{2\pi} \sum_{if} Q_{ii} |x_{if}|^2 \Omega_{if}^3 R_s^\tau(\vartheta, \varphi) B_s^\tau(\omega_{if}) \tag{11}$$

where

$$B_s^\tau(\omega_{if}) \equiv \sum_{\mu=1,2} |\xi_{\mu s}^\tau(\omega_{if})|^2$$

$$R_\pi^\tau(\vartheta, \varphi) = \left(\frac{v_\parallel \sin\vartheta \cos\varphi [\cos\vartheta (\boldsymbol{n}_1 \boldsymbol{\tau}) - \tau_z]}{|\boldsymbol{\tau}_\perp| (1 - v_\parallel \cos\vartheta)^2} + \frac{\sin\vartheta \cos\varphi (\boldsymbol{n}_1 \boldsymbol{\tau}) - \tau_x}{|\boldsymbol{\tau}_\perp| (1 - v_\parallel \cos\vartheta)^2} \right)^2$$

$$R_\sigma^\tau(\vartheta, \varphi) = \left(\frac{v_\parallel \sin^2\theta \cos\varphi (\tau_y \cos\varphi - \tau_x \sin\varphi)}{|\boldsymbol{\tau}_\perp| (1 - v_\parallel \cos\vartheta)^2} + \frac{\tau_z \sin\vartheta \sin\varphi - \tau_y \cos\vartheta}{|\boldsymbol{\tau}_\perp| (1 - v_\parallel \cos\vartheta)} \right)^2$$

$$x_{if} = \int_0^a \varphi_{i\kappa}(x) x \varphi_{f\kappa_1}^*(x)\, dx \qquad \boldsymbol{n}_1 = \frac{\boldsymbol{k}_\tau}{|\boldsymbol{k}_\tau|}$$

and ϑ is the angle between the vector $\boldsymbol{k}_\tau = \boldsymbol{k} + 2\pi\boldsymbol{\tau}$ and the z axis.

(*b*) For radiation propagating along the direction of motion of the channelled particle:

$$\frac{dN_s^0}{d\Omega} = \frac{e^2 L}{2\pi} \sum_{if} Q_{ii} |x_{if}|^2 \Omega_{if}^3 R_s^0(\theta, \psi) B_s^0(\omega_{if}) \tag{12}$$

where

$$B_s^0(\omega_{if}) \equiv \sum_{\mu=1,2} |\xi_{\mu s}^0(\omega_{if})|^2$$

$$R_\sigma^0(\theta, \psi) = \left(\frac{v_\parallel \sin^2\theta \cos\psi (\tau_y \cos\psi - \tau_x \sin\psi)}{|\boldsymbol{\tau}_\perp| (1 - v_\parallel \cos\theta)^2} + \frac{\tau_z \sin\theta \sin\psi - \tau_y \cos\theta}{|\boldsymbol{\tau}_\perp| (1 - v_\parallel \cos\theta)} \right)^2$$

$$R_\pi^0(\theta, \psi) = \left(\frac{\sin^2\theta \cos\psi (\tau_x \cos\psi + \tau_y \sin\psi)}{|\boldsymbol{\tau}_\perp| (1 - v_\parallel \cos\theta)^2} + \frac{\tau_z \sin\theta \cos\psi - \tau_x}{|\boldsymbol{\tau}_\perp| (1 - v_\parallel \cos\theta)} \right)^2$$

and θ is the angle between the vector $\boldsymbol{k}$ and the z axis; φ and ψ are the azimuthal angles which are measured from the x axis.

The approximate integral expressions for the number of γ quanta emitted within the diffraction peak can easily be obtained by making use of the following circumstance: the frequency of the photon diffracted by a crystal and the position of the diffraction peak are defined not only by the Bragg condition but, at the same time, by conservation laws which are satisfied by a radiation process of an oscillator with transition frequency Ω_{if}. As a result, we obtain the following expressions for the total number of γ quanta emitted within the diffraction peak at a large angle relative to the direction of motion of the channelled particle:

(*a*) For π polarisation:

$$\Delta N_\pi^\tau \simeq \frac{\pi e^2 L \beta_1^2 \tau^4 |g_{00}'|}{8\tau_z^2 |\tau_z|} \sum_{if} Q_{ii} |\chi_{if}|^2 \Omega_{if}^2 \left[\frac{\tau_x^2}{\tau_\perp^2} + \frac{\pi\tau^2 |\tau_z| \Omega_{if}}{\tau_\perp^2 \omega_{ifm}^2} \left(1 - \frac{\pi\tau^2}{2|\tau_z| \omega_{ifm}} \right) \right] \tag{13}$$

$$\omega_{ifm} \simeq 2\Omega_{if} E^2 / m^2.$$

(*b*) For σ polarisation:

$$\Delta N_{\sigma}^{\tau} \simeq \frac{\beta_1^2 e^2 L\pi\tau^4 |g'_{00}|}{8\tau_z^2|\tau_z|} \sum_{if} Q_{ii}|\chi_{if}|^2\Omega_{if}^2\left[\frac{2\tau_y^2}{\tau_\perp^2} + \frac{\tau_x^2-\tau_y^2}{\tau_\perp^2}\left(1-\frac{\pi\tau^2}{|\tau_z|\,\omega_{ifm}}\right)\right]. \quad (14)$$

To investigate the possibility of using the radiation considered above as a source of x-rays and γ radiation with easily variable frequency, we estimate the total number of γ quanta in the diffraction peak. We notice that equation (11) can be expressed as a product of the radiation spectrum in the absence of diffraction and the function $B^{0,\tau}_{\sigma,\pi}(\omega_{if})$ characterising the photon reflection in the crystal at the diffraction condition. The value of the function is of the order of unity in the vicinity of the angle $\sin\vartheta\Delta\vartheta \sim |g'_{00}|\Omega_{if}/2|\tau_z|v_{\parallel}$ if the Bragg condition is satisfied and it quickly goes to zero when deviating from the diffraction condition. It follows that the angular value of the maximum radiation intensity in diffraction is determined by the structure amplitudes such as those in the case of the dynamic diffraction of γ quanta in the crystal. Hence, the angular value of the diffraction peak of the radiation intensity is $\Delta\vartheta \sim 10^{-6}$ rad which is much less than the Lindhard angle characterising the channelling process of the particle. Thus, the total number of γ quanta radiated within the diffraction peak is approximately the number of γ quanta radiated in the absence of diffraction within the angular divergence $\Delta\vartheta \sim 10^{-6}$ rad near the maximum intensity. The formula for the estimation can be written in the form $\Delta N_{\omega}^{\tau} \sim e^2 L\chi_0^2\Omega^4\gamma^4|g'_{00}|/|\tau_z|$. It follows from the above estimate that the particle can radiate $N_{\omega} \sim 10^7$ γ quanta per second for a particle energy $E \sim 30$ MeV and a particle current of 10^{-6} A within the angle $\Delta\vartheta \sim 10^{-6}$ rad. This value essentially exceeds the intensity of an ordinary x-ray source in the same angular and spectral intervals.

5. Conclusion

A diffraction pattern can be always observed for x-rays formed by channelled particles. This means that in addition to the photons propagating in a forward direction there is a diffraction peak of radiation at a large angle relative to the particles direction of motion. To identify such a diffraction pattern of radiation observed along the z axis it is necessary to measure the spectral distribution with high precision because the diffraction produces a redistribution of the intensity in the radiation spectrum.

In conclusion, it should be noted that the advantages of such diffraction experiments for channelling radiation are, firstly, in the high intensity of the radiation from the channelled particle, and secondly, in the possibility of receiving a comprehensive diffraction pattern without any loss in the radiation intensity which would be present if the channelling radiation were used as the usual external radiation source for subsequent diffraction measurements.

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